# fish5107stockpred Prediction of stock and catch 

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## 1 Introduction

### 1.1 Forward prediction of a stock

Given an assessment one can predict the future stock
Need to know future recruitment
Need to determine catches (quotas or $F$ etc.)
Assumptions on $M$, mean weight, etc.

### 1.1. 1 Details

It is a fairly simple exercise to use outcomes from a stock assessment to predict the development of the fish stock given assumptions on recruitment and catches. However, there are several details which need to be addressed.

### 1.1.2 Examples

Example 1.1. Haddock in Icelandic waters. Data for predictions, taken from Marine Research Institute assessment 2011 report.
http://tutor-web.net/fish/fish5107stockpred/lecture10/iceland-haddock-data.r

### 1.2 Simulating initial conditions



### 1.2.1 Details

In many cases it is desirable to simulate a stock forward in time from some historical time point and subsequently compare the results with data. Alternatively one may simply want to test the effects of different fishing mortalities on catches, stock parameters and so forth.

In such cases it is useful to be able to simulate the initial stock size in numbers at age.

If this stock has been historically fished for a long time at the same fishing mortality, then it has reached an equilibrium at some given spawning stock biomass. However, since this is an equilibrium, the stock also has an equilibrium age composition.

Such a stock provides a useful tool for investigating the effects of different forward projection scenarios.

### 1.2.2 Examples

Example 1.2. Consider simulating a stock which has Beverton-Holt stock-recruit parameters $\alpha$ and $K$, a given natural mortality $M$, a selection pattern, maturity, and weight at age.

If R is used to set up the appropriate data and functions, the following commands may be used to generate the age compositions using the simulated fish stock.

Simulated fish stock:
http://tutor-web.net/fish/fish5107stockpred/lecture10/simdat.r
Age composition code:
http://tutor-web.net/fish/fish5107stockpred/lecture10/agecomp.r

### 1.3 Carrying forward stock numbers from an assessment

An assessment gives stock numbers at the beginning of the last data year.
First project to the end of the data year.

Note that mortalities are also available for the data year.

$$
N_{a y}=N_{a-1, y-1} e^{-Z_{a-1, y-1}}
$$

### 1.3.1 Details

Suppose the age composition of the stock at the beginning of a specific year is given, either from an assessment or simply as a set of simulated numbers. For such a set of stock numbers at age it is of course simple to compute catch projections.

An assessment commonly provides the number of fish alive in the sea at the beginning of the last year for which data is available. If catch projections are required for a future year, the stock size must first be updated to the beginning of the last data year by using the stock projection equation.

Definition 1.1. Stock projection equation:

$$
N_{a y}=N_{a-1, y-1} e^{-Z_{a-1, y-1}}
$$

It is possible to use the stock project equation since the mortality coefficients for the last year are a part of the assessment. Thus, the stock size at the beginning of the next fishing year is known, except for the recruiting year-class.
For the youngest age, i.e. recruitment, either an average or other method such as a survey index is used to complete the forward projection.

It should be noted that sometimes the catch from a few of the youngest age groups has been relatively small and therefore little true knowledge has been obtained on these yearclasses and the extrapolation becomes imprecise. When this is the case, survey indices are sometimes used to predict a few more age groups in addition to the youngest one.

### 1.3.2 Examples

Example 1.3. Consider again the haddock in Icelandic waters. Look at all of the input data together
inpdat<-cbind(w, ws, p, Fmort, natmort, n2010, c2010)
dimnames (inpdat) [[1]]<-ages
A simple forward projection in R just uses the code
n2010*exp (- (Fmort+natmort))
The projections need to be done in two stages, first simple forward computations which generate ages 3 through 10+
n.temp<-n2010*exp(-(Fmort+natmort))

We need to rearrange to put in the recruitment at the front and add the last two ages to set up the plus group correctly.
n2011<-c (Recr, n. $\operatorname{temp}[1:(A-1)], n . \operatorname{temp}[A]+n . \operatorname{temp}[A+1])$

### 1.4 Fishing mortality assumptions

Future predictions can use

$$
F_{a y}=F_{a, y-1}
$$

or

$$
F_{a y}=F_{y} s_{a}
$$

### 1.4.1 Details

In addition to the stock size estimation, an assessment also gives estimates of fishing mortality for the last data year. When projecting the stock forward in time assumptions need to be made on fishing mortality in the future.

The simplest catch projection assumes a constant fishing mortality into the future.

Definition 1.2. Catch projection with constant fishing mortality:

$$
F_{a y}=F_{a, y-1}
$$

A special case in which a constant selection pattern is assumed and only the overall fishing mortality is determined also exists.

Definition 1.3. Catch project with constant selection pattern:

$$
F_{a y}=F_{y} s_{a}
$$

$s_{a}$ is the selection pattern obtained from the assessment

The level of overall fishing mortality can be determined by several means (the simplest of which is to set it to a constant in the future).

### 1.4.2 Examples

Example 1.4. When simulating a stock one has the stock size in numbers at age at the beginning of the prediction period. One then needs to set the future fishing mortality and natural mortality.

Typically one will continue to use the historical selection pattern when predicting into the future, as well as a constant natural mortality.

One then needs only to define the fishing mortality to be used in the future. This will usually correspond to a specific strategy or an exploration of different scenarios.

In $R$ the prediction is started by setting up the fishing and total mortality vectors:
NO<-Nhist
Fmort<-0. 25
Z<-Fmort*sa+M

### 1.5 Predicting the catch

Catch prediction

$$
\begin{gathered}
C_{a y}=\frac{F_{a y}}{Z_{a y}}\left(1-e^{-Z_{a y}}\right) N_{a y} \\
Y_{y}=\sum_{a} w_{a y} C_{a y}
\end{gathered}
$$

### 1.5.1 Details

## Definition 1.4. Catch by numbers in each age group:

$$
C_{a y}=\frac{F_{a y}}{Z_{a y}}\left(1-e^{-Z_{a y}}\right) N_{a y}
$$

Definition 1.5. Total catch in tonnes:

$$
Y_{y}=\sum_{a} w_{a y} C_{a y}
$$

The computation of catch with a constant fishing mortality is therefore very simple. It is also fairly easy to predict certain proportional increases or decreases in effort and the effects of conservation measures can also be specified.

### 1.5.2 Examples

Example 1.5. Continuing with the haddock example, the catch can be computed using:
canum2011<-(Fmort/(Fmort+natmort) $) *(1-\exp (-($ Fmort+natmort $))) *$ n2011
Y2011<-sum (canum2011*w)
Y2011

### 1.6 Short-term predictions: Assumptions

Short-term assumptions:

- Current stock size
- Recruitment
- Mean weights
- Selection pattern
- Annual $F$

Often take uncertainty into account - mainly in current stock size

### 1.6.1 Details

The first predictions for a stock usually consist of short-term projections on stock development, typically $1-5$ years.

This is based on the same equations as before and uses the same fishing mortality coefficient as was used for the catch projection. Thus, it is possible to predict catch and stock development a few years ahead, or as far into the future as exiting information on recruitment permits.

The assumptions in these predictions include:

- current stock size
- recruitment
- mean weights
- selection pattern
- annual $F$

In order to obtain some estimate of uncertainty one needs to incorporate the uncertainty in some of these assumptions. In particular, the uncertainty in the current stock size needs to be taken into account.

### 1.7 Projecting the stock in numbers and biomass

$$
\begin{gathered}
N_{a y}=N_{a-1, y-1} e^{-Z_{a-1, y-1}} \\
S_{y}=\sum_{a} w_{a} N_{a y}
\end{gathered}
$$

### 1.7.1 Details

As described earlier, the stock projection is implemented using the stock projection equation:

$$
N_{a y}=N_{a-1, y-1} e^{-Z_{a-1, y-1}}
$$

When predicting, it must be noted that an assessment commonly only provides the stock size in numbers at the beginning of the last data year, whereas for prediction one needs these at the end of the last data year. This projection is done using the stock equation based on the fishing mortality coefficients as obtained from the assessment. Thus, the stock size at the beginning of the first prediction year can be assumed known, except for the recruiting year-class.

For the youngest age, i.e. recruitment, either an average or other method, such as a survey index, is used.

It should be noted that sometimes the catch from a few of the youngest age groups has been relatively small and therefore little true knowledge has been obtained on these year-classes and the above projection across the last data year becomes imprecise. When this is the case, survey indices are sometimes used to estimate a few more age groups in addition to the youngest one.

The projections into the future are now carried out using the stock equation and the fishing mortalities described earlier.

The spawning biomass is computed as usual.
Incoming recruitment each year is usually estimated from survey data, set at average values or a stock-recruitment function is used.

### 1.7.2 Examples

Example 1.6. Consider the simulated population again, with the start of year population in $N_{0}$.

```
S<-sum(wa*pa*N0)
N1<-NO*exp(-Z)
```

This leaves $N_{1}$ missing the youngest and having an age which is older than the oldest age in $N_{0}$ and this needs to be corrected using either a plus-group or dropping the oldest.

Here, we drop the oldest and insert recruitment from the stock-recruitment function:

```
R<-(alpha*S/(1+S/K)) # Recr without variation
NO<-c(R,N1[1:(length(N1)-1)])
```


### 1.8 Other details in predictions

```
Predict \(w_{a y}\) ?
Predict \(p_{a y}\) ?
```


### 1.8.1 Details

No mention has been made of how one can best project mean weight at age ( $w_{a y}$ ) or other similar quantities into the future. There are several models available, but it is of course simplest to assume the same mean weight next year as was measured this year (or use some historical average). Sometimes, additional information is available and it may be necessary to predict mean weight by use of some model. Such a model may potentially take into account availability of food, temperature, stock size, etc. as applicable.

The same considerations apply to the proportion mature at age each year into the future, $p_{a y}$.

It is a fairly general rule, that the variation in quantities such as mean weight at age and maturity at age are of secondary importance compared to the overall fishing mortality which is the driving factor in over-fished systems.

### 1.9 Short-term predictions

Short-term assumptions:

- Current stock size
- Recruitment
- Mean weights
- Selection pattern
- Annual $F$

Often take uncertainty into account - mainly in
 recruitment and current stock size

### 1.9.1 Details

The first prediction step consists of short-term projections on stock development, typically $1-5$ years.

This is based on the same equations as before and uses the same fishing mortality coefficient as was used for the catch projection. Thus, it is possible to continue predicting catch and stock development a few years ahead, or as far into the future as exiting information on recruitment permits.

In order to obtain some estimate of uncertainty one needs to incorporate the uncertainty in some of these assumptions. In particular, the uncertainty in the current stock size and future recruitment needs to be taken into account.

### 1.9.2 Examples

Example 1.7. Use the same data for haddock in Icelandic waters to conduct the following calculations:

Icelandic Haddock Data:
http://tutor-web.net/fish/fish5107stockpred/lecture10/iceland-haddock-data.r
Short-term Prediction Calculations:
http://tutor-web.net/fish/fish5107stockpred/lecture10/haddockshorttermprediction.r

### 1.10 Medium-term predictions

| First-year stock size |
| :--- |
| Recruitment: Use S-R relationship |
| Mean weights |
| Selection pattern |
| Annual $F$ |
| Uncertainty needs to be incorporated |

### 1.10.1 Details

Naturally the uncertainty increases as the predictions are carried further into the future. It is possible to use results from stock-recruitment estimation to obtain bounds on this uncertainty. For such medium-term prediction the uncertainty increases even further since some relation between spawning stock biomass and recruitment needs to be used.

### 1.11 Target assumptions - harvest control rule

For medium-term need to assume some target, e.g.
$F=F_{0.1}$
$F=F_{\text {max }}$
or other harvest control rule

### 1.11.1 Details

The above methods can be used to make projections regarding catch and stock development given assumptions on fishing mortality. It is, for example, easy to project the stock and catch based on $F_{0.1}, F_{\max }$ etc.

The effects of a rule which specifies a catch quota are slightly more difficult to compute, since this requires the estimation of a fishing mortality rate corresponding to the catch quota.

## 2 The issue of harvest control laws

### 2.1 Introduction

Any prediction needs to take into account future catches.
How these are set defines a catch control law.

This applies to short, medium, and long-term predictions.

### 2.2 Implementable control rules

### 2.2.1 Details

Note 2.1. A harvest control rule is here defined as a mathematical formula incorporating biological information to give a quota, target effort or closed areas etc.

Within the International Whaling Commission a great deal of work has been put into developing such rules and similar work has also been undertaken under the auspices of other international bodies.

Some rules are easy to state but impossible to implement. The effect, for example, of decreasing fishing mortality on a given species to $F_{M S Y}$ over a period of about 5 years, is easy to test out. This is an example of a simple harvest control rule that lends itself perfectly to statistical testing but is, unfortunately, impossible to put into practice since all of the quantities involved are only known with considerable uncertainty.

A corresponding rule, which can be implemented is: First estimate both $F_{M S Y}$ and current fishing mortality, $F$, and set the coming years quota to that predicted by $0.8 F+0.2 F_{M S Y}$, followed by $0.6 F+0.4 F_{M S Y}, 0.4 F+0.6 F_{M S Y}, 0.2 F+0.8 F_{M S Y}$ and finally to $0 F+1.0 F_{M S Y}$. Testing this rule needs to take into account the uncertainty in the estimate of current fishing mortality.

Similarly, one can implement several nonparametric rules, such as: Reduce $Q$ by $15 \%$ if CPUE is high and increase if CPUE is low. Maintain catch if CPUE is within some range.

Note that all of the evaluations of these procedures must take into account situations where quotas are exceeded or management does not work in some way.

### 2.2.2 Examples

Example 2.1. The following is an example of implementable harvest control rule: Stock size estimations are carried out each year (according to a precise formula). If the spawning stock is estimated at below 500 thousand tonnes, the catch compared to previous year's catch is reduced by $15 \%$. If the estimation is above 800 thousand tonnes, the catch is increased by $15 \%$. If the spawning stock is estimated at between 500 and 800 thousand tonnes, there are no changes in the catch.

Such rules are of little use if nothing is known about how they work in practice. It is, however, possible to examine how the rule could have worked in the past and simulate its future behavior. When such tests are carried out, all influential factors must be considered. For example, if the fishery is managed in such a way that the catch always exceeds the quota by $10 \%$, this excess must be taken into account when the rule is tested.

### 2.3 Implementation

### 2.3.1 Details

Implementing a harvest control rule is always difficult. In particular the decision to apply a rule entails a decision to comply with that particular rule, so that political decision making is thereby entered into a formula.

It turns out that in many real situtations it is possible to come to a decision on a long-term strategy, but it is more difficult to decide to reduce catches in the short-term. In such a situation it is sometimes possible to decide on a long-term strategy which goes smoothly from the current catches or fishing mortality into a more satisfactory harvest regime in the medium-term.

If the stocks are in really poor shape then it may not be realistic to continue fishing for several years at the same fishing mortality rates.

## 3 Short-term prediction

### 3.1 Issues in short-term predictions

Issues in making short-term predictions:

- Initial stock size
- Recruitment


### 3.1.1 Details

A short-term prediction will normally be driven by the assumptions on the initial size and age-composition of the stock.

The recruitment may, however, dominate the predictions if the species is short-lived. This applies also to severely over-fished, long-lived species in which case even the first predicted year may depend heavily on the assumed or estimated recruitment in the first year or two of the prediction.

### 3.2 Goals with short-term predictions

Goals with short-term prediction:

- Predict catch and stock development
- Identify a catch level which does not reduce the stock

These should be completed with $90 \%$ probability.


Comparing catch and stock projection for F from 0 to 1 .

### 3.2.1 Details

In the simplest case one wants to predict catch and stock development in the short-term. This is, however, quite short-sighted. Normally one also wants, for example, to find the catch level which does not reduce the stock from its current size or satisfies some such criterion.

In recent years it has become common practice to require such a criterion to be fulfilled with some specified probability. When the uncertainty is excluded, the approach really corresponds to finding the catch level which satisfies the criterion with $50 \%$ probability. When put this way, it is usually not appropriate.

### 3.2.2 Examples

Example 3.1. The figure shows how a variety of short-term predictions are typically included in a single figure, comparing the effects of different levels of fishing mortality.

### 3.3 Adding uncertainty

| Assessment uncertainty |  |
| :--- | :--- |
| Recruitment uncertainty |  |

### 3.3.1 Details

For short-term predictions one needs to consider the effect of uncertainty on the stock estimate.

If the stock has only a few year-classes then the incoming recruitment may have considerable impact in the short-term, in which case one needs to consider recruitment variation as well.

### 3.3.2 Examples

Example 3.2. Short-term predictions with R:
The figure gives an example of how a short-term prediction for a fixed fishing mortality can be augmented by including errors in initial assessment, recruitment variation and annual implementation errors.

The wide, dashed lines indicate percentiles $(0,25,50,75$ and 100 ) whereas the thin lines are five examples of simulated trajectories.

## 4 Medium-term predictions

### 4.1 Variability

### 4.1.1 Details

For obvious reasons, the testing of any harvest control rule is insufficient if uncertainty factors are not taken into account. There are two primary sources of uncertainty, measurement uncertainty and future environmental effects.
Note 4.1. Measurement uncertainty refers to uncertainty such as the current stock estimate or natural mortality.

The relationship between spawning stock and recruitment is also uncertain.
Variable environmental conditions will lead to variation in recruitment in the future, just as they have done in the past.

### 4.2 Estimating recruitment variability

### 4.2.1 Details

Variation around the stock-recruitment curve will tend to dominate the variability in mediumterm predictions. This variation can be estimated from the fitted stock-recruit function using the variation estimation equation.

## Definition 4.1. Variation estimation equation

$$
\sigma^{2}=\frac{\sum_{y}\left(\ln \left(R_{y}\right)-\ln \left(\hat{\alpha} S_{y} e^{-S_{y} / \hat{K}}\right)\right)^{2}}{n-2}
$$

The variation estimate equation works if a Ricker curve has been fitted as well as for other stock-recruit functions.

Future recruitment can now be simulated by generating $\ln (R)$ from a Gaussian distribution with this standard error and a mean as specified by the Ricker curve. Note that this ignores errors in the parameter estimates.

### 4.3 Including variability in predictions



### 4.3.1 Details

Including recruitment variation is fairly simple. Normally lognormal errors are assumed around a stock-recruit curve.

Assessment errors also need to be included, particularly for a short time interval.

### 4.4 Presenting medium-term predictions



### 4.4.1 Details

Medium-term predictions are commonly presented in summary graphs. Typically a few example trajectories and quantiles are presented.

## 5 Linking assessments and all projections

### 5.1 Linking assessments and all projections



### 5.1.1 Details

The medium-term projection is a natural extension of the short-term projection.
The equilibrium yield and biomass computations correspond to an infinite-horizon continuation of the medium-term projection.

It is often useful to include the following on a single plot with SSB on the x -axis and yield on the $y$-axis:

- the current level of biomass and yield
- the equilibrium biomass-yield curve
- the projection from the current point through the projections towards the equilibrium
- the historical trend of (S,Y)-pairs


## 6 Case study

### 6.1 Catch Rules for cod in Icelandic waters

### 6.1.1 Details

An example of harvest control rules for cod in Icelandic waters can be found in an article by Friðrik Baldursson et.al. (1993). Examples processed in a similar way will be demonstrated below.

One type of harvest control rules is based on fixing a quota which is dependent on a specific stock size. Thus, it is possible to visualize an increase in catch along with an increase in stock size. It is obvious that an unlimited and uncontrolled increase in the catch would never be feasible. Deviations in measurements might result in an excessive catch during one particular season, which in turn would make it imperative to keep catches very small for a long time to follow. It is, therefore, only sensible to fix an upper limit for the catch and never allow it to exceed a certain maximum.

Various managerial arguments may be put forward to the effect that it will be impossible to decrease the catch below a certain minimum. In reply to this, it should be noted that this minimum will never be caught if the stock is smaller than the actual catch and thus the catch decreases in spite of all attempts to keep it above the minimum.

When a harvest control rule is examined, it is a good idea to start by observing how it works when there are no measurement deviations or environmental fluctuations that confuse the issue.

### 6.2 Catch Quota Rules for cod in Icelandic waters

### 6.2.1 Details

The size of the spawning stock over the period of 1992-1993 is estimated at 200 thousand tonnes. At this spawning stock size, the catch could amount to 200 thousand tonnes without putting the equilibrium of the stock at risk in the long run. The harvest control rule, on the other hand, stipulates a smaller catch for this stock size and not in excess of 175 thousand tonnes. As a result, the stock will continue to grow as long as the curve of the harvest control rule is below the curve of equal balance. After such an increase, the stock is likely to give a greater yield.

By using this rule (and all the underlying assumptions) the harvest control rule will in the end give the same catch as the equal balance curve when the spawning stock has reached 700 thousand tonnes. If the stock grows more than this, the harvest control rule will take more than the equal balance catch and the stock will decrease again towards the point of intersection.

A survey of harvest control rules is more complex than a simple stock size estimation. This is not only a question of finding out the equal balance catch. It will also be necessary to assess how long it will take for the stock to reach equilibrium, what the total catch will be, and finally this information needs to be compared for various different rules. Then the effects of changes in the main assumptions of the computations need to be examined.

### 6.3 Variability

### 6.3.1 Details

The above-mentioned examinations can be carried out in a relatively simple manner as an addition to a more traditional catch projection by the use of simulation models where variability is inserted in order to simulate changes in environmental conditions. The typical results from one simulation then correspond to the effects of certain environmental conditions on the stock and the catch. One simulation thus provides examples of possible developments of stock and catch for a few years into the future. Other simulations correspond to other possibilities for assumptions or environmental conditions. About 100 simulations of a catch rule should give a survey of what kind of development is likely and what is unlikely to happen.

For further information on this subject, interested readers are referred to the article mentioned above, where various assumptions and the effects they might have, e.g. on the probability of collapse, are examined.

