

Multivariate probability distributions

math612.0 From numbers through algebra to calculus and linear algebra

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Joint probability distribution

If

X_1, \dots, X_n are discrete random variables with

$P[X_1 = x_1, X_2 = x_2, \dots, X_n = x_n] = p(x_1, \dots, x_n)$, where x_1, \dots, x_n are numbers, then the function p is the joint probability mass function (p.m.f.) for the random variables X_1, \dots, X_n .

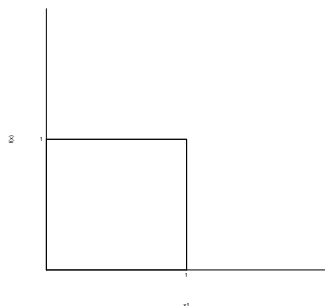
For continuous random variables Y_1, \dots, Y_n , a function f is called the joint probability density function if,

$$P[Y \in A] = \int \int \dots \int f(y_1, \dots, y_n) dy_1 dy_2 \dots dy_n.$$

The random sample

A set of random variables X_1, \dots, X_n is a random sample if they are independent and identically distributed (i.i.d.).

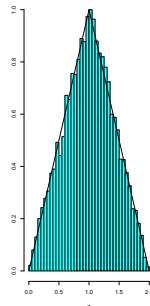
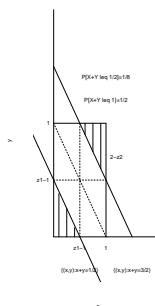
A set of numbers x_1, \dots, x_n are called a random sample if they can be viewed as an outcome of such random variables.



The sum of discrete random variables

The sum of two continuous random variables

If X and Y are continuous random variables with joint p.d.f. f and $Z = X + Y$, then we can find the density of Z by calculating the cumulative distribution function.



Means and variances of linear combinations of independent random variables

If X and Y are random variables and $a, b \in \mathbb{R}$, then

$$E[aX + bY] = aE[X] + bE[Y].$$

Means and variances of linear combinations of measurements

If x_1, \dots, x_n and y_1, \dots, y_n are numbers, and we set

$$z_i = x_i + y_i$$

$$w_i = ax_i$$

where $a > 0$, then

$$\bar{z} = \frac{1}{n} \sum_{i=1}^n z_i = \bar{x} + \bar{y}$$

$$\bar{w} = a\bar{x}$$

$$s_w^2 = \frac{1}{n-1} \sum_{i=1}^n (w_i - \bar{w})^2$$

$$= \frac{1}{n-1} \sum_{i=1}^n (ax_i - a\bar{x})^2$$

The joint density of independent normal random variables

If $Z_1, Z_2 \sim n(0, 1)$ are independent then they each have density

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, x \in \mathbb{R}$$

and the joint density is the product $f(z_1, z_2) = \phi(z_1)\phi(z_2)$ or

$$f(z_1, z_2) = \frac{1}{(\sqrt{2\pi})^2} e^{-\frac{z_1^2}{2} - \frac{z_2^2}{2}}.$$

More general multivariate probability density functions