

# STATS510.stats 510 10 Odds ratios and relative risk

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# 1 Introduction to odds ratios and relative risk

## 1.1 Binary response

A **Binary Response** is a variable that has only two possible outcomes.

Binary responses are common in life sciences.

Examples:

- Yes / No
- Success / Failure
- Sick / not sick
- Fractured hip / hip not fractured

Example of the occurrence of two binary responses, smoking and plaque.

	plaque	no plaque	total
Smoker	39	171	210
Non-smoker	73	679	752

Data: 35 to 70 year old men, not on statin-treatments from Hjartavernds risk factor survey 2006-2007. Results from a questionnaire and “hálsæðaðómun” at arrival.

## 1.2 Odds

Binary responses are commonly described with *odds*.

The **Odds of an outcome**, is the probability that the outcome occurs divided by the probability that it does not occur. If the probability of the outcome is  $p_1$ , then

$$\text{Odds} = \frac{p_1}{1-p_1}$$

- The proportion of smokers with plaque:

$$p_1 = 39/210 = 0.186 \quad \text{or} \quad 18.6\%$$

- The proportion of non-smokers with plaque:

$$p_0 = 73/752 = 0.097 \quad \text{or} \quad 9.7\%$$

- Odds of plaque with smokers:

$$\frac{p_1}{1-p_1} = \frac{0.186}{1-0.186} = 0.229$$

- Odds of plaque with non-smokers:

$$\frac{p_0}{1-p_0} = \frac{0.097}{1-0.097} = 0.107$$

### 1.3 Relative Risk

The **Relative Risk** of an outcome in two populations is the proportion of the probability of the outcome in the two populations. If we denote the probabilities with  $p_0$  and  $p_1$  then

$$\text{Relative Risk} = \frac{p_1}{p_0}$$

Commonly, the probability  $p_1$  corresponds to the probability of the outcome in a population of interest, whereas  $p_0$  corresponds to the probability in a reference population.

	plaque	no plaque	total
Smoker	39	171	210
Non-smoker	73	679	752

Data: 35 to 70 year old men, not on statin-treatments from Hjartavernds risk factor survey 2006-2007.

The relative risk of plaque for smokers, compared to non-smokers:

$$RR = \frac{p_1}{p_0} = \frac{0.186}{0.097} = 1.91$$

### 1.4 Interpretation of Relative Risk

itemize

- If  $RR = 1$  the risk is equal in the two populations.
- If  $RR < 1$  the risk is less in the population of interest than in the reference population.
- If  $RR > 1$  the risk is greater in the population of interest than in the reference population.

	plaque	no plaque	total
Smoker	39	171	210
Non-smoker	73	679	752

Data: 35 to 70 year old men, not on statin-treatments from Hjartavernds risk factor survey 2006-2007.

The relative risk of plaque for smokers, compared to non-smokers is

$$RR = \frac{p_1}{p_0} = \frac{0.186}{0.097} = 1.91$$

The relative risk is greater than 1 which shows that the risk is greater for smokers than non-smokers.

	outcome occurs	outcome does not occur	total
Population of interest	a	b	a+b
Reference population	c	d	c+d

## 1.5 Odds Ratios

Odds Ratios (OR) can be used as a measure of correlation with a binary response.

The **Odds Ratio** of an outcome in two populations is the proportion of the odds of the outcome in the two populations. If we denote the probabilities with  $p_0$  and  $p_1$  then

$$\text{Odds Ratio} = \frac{\frac{p_1}{1-p_1}}{\frac{p_0}{1-p_0}}$$

Commonly, the probability  $p_1$  corresponds to the probability of the outcome in a population of interest, whereas  $p_0$  corresponds to the probability in a reference population.

When odds ratios are calculated, the results are often represented in a table. With this representation the **odds ratio**

is:

$$\frac{ad}{bc}$$

	plaque	no plaque	total
Smoker	39	171	210
Non-smoker	73	679	752

Data: 35 to 70 year old men, not on statin-treatments from Hjartavernds risk factor survey 2006-2007.

$$\text{Odds ratio} = \frac{39 \cdot 679}{73 \cdot 171} = 2.14$$

## 1.6 Interpretation of Odds Ratios

Odds ratios are interpreted in a similar manner as relative risk.

**Itemize**

If OR = 1 the odds are equal in the two populations.

If OR < 1 the odds are less in the population of interest than in the reference population.

If OR > 1 the odds are greater in the population of interest than in the reference population.

	plaque	no plaque	total
Smoker	39	171	210
Non-smoker	73	679	752

Data: 35 to 70 year old men, not on statin-treatments from Hjartavernds risk factor survey 2006-2007.

A smoker has higher odds for plaque than a non-smoker.

$$\text{Odds ratio} = \frac{39 \cdot 679}{73 \cdot 171} = 2.14$$

The OR is  $> 1$

There seems to be positive correlation between smoking and plaque.

## 1.7 Advantages of Odds Ratios

- Odds Ratios can always be interpreted, independent on the study design:
  - Cohort
  - Cross sectional
  - Case-control
- Binary responses are common in life sciences
- Odds Ratios can easily be estimated with logistic regression.

## 1.8 Odds Ratios as a measure of Relative Risk

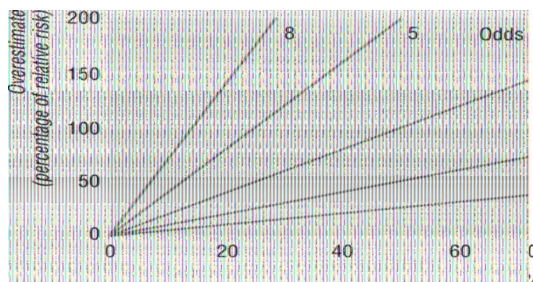


Figure 1: Amount by which odds ratios of  $>1$  overestimate relative risk, for different odds ratios and different levels of initial risk. Davies, H. T. O. et al. BMJ 1998;316:989-991

- Notice that if  $RR > 1$  then  $OR > RR$
- When the probability of an outcome in the reference population is less than 20%, OR in the range of 1.5 to 2 can be interpreted as a corresponding increase in risk.
- According to the figure, the risk is overestimated by approximately 20%, but one keeps that in mind.

- 2 Odds ratios and relative risk with R**
- 3 Maximum likelihood estimation**
- 4 kx2 tables**
- 5 Mantel Haenszel method**