# Matrix representation of SLR stats544-1-slr Applied simple linear regression 

Gunnar Stefansson

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## Introduction

It is convenient to represent the multiple linear regression model in matrix form. For that we need:

- Matrices
- Matrix operations
- Matrix representation of SLR


## Purpose of matrix representation

It is easy to set up matrices which describe the simple linear regression model. Solving this using matrix algebra gives an alternative representation of the estimators.

## Matrix form of simple linear regression

$y \in R^{n}=$ vector of measurements

$$
\mathbf{X}=\left[\begin{array}{rr}
1 & x_{1} \\
\vdots & \vdots \\
1 & x_{n}
\end{array}\right]
$$

the "X-matrix"
$\min \sum\left(y_{i}-\left(\beta_{0}+\beta_{1} x_{i}\right)\right)^{2}$ is equivalent to finding

$$
\boldsymbol{\beta}=\binom{\beta_{0}}{\beta_{1}}
$$

to mininmize $\|\mathbf{y}-\mathbf{X} \boldsymbol{\beta}\|^{2}$
Number notation: $\mathbf{y}=\mathbf{X} \boldsymbol{\beta}+\mathbf{e}$

## Prediction as linear projection



Geographical representation of linear model.
Basic model: $\mathbf{y}=\mathbf{X} \boldsymbol{\beta}+\mathbf{e}$ where $\mathbf{X}$ is a matrix of dimensions $n \times p$ (here $p=2$ ).
Closest prediction of $\mathbf{y}$ within the model is via orthogonal projects of $\mathbf{y}$ onto the plane spanned by the column vectors of $\mathbf{X}$.
Predicted values: $\hat{\mathbf{y}}$.
The projection is in $s p\{\mathbf{X}\}$, so it is a linear combination of column vectors of $\mathbf{X}$ so we can write

$$
\hat{\mathbf{y}}=\mathbf{X} \hat{\boldsymbol{\beta}}
$$

## Geometric solution to the simple linear regression problem

From linear algebra the matrix solution is known

$$
\hat{\boldsymbol{\beta}}=\mathbf{X}^{\prime} \ldots
$$

and also know $\hat{\beta}=\sum \ldots \hat{\beta}_{0}=\hat{y}-\hat{\beta}_{1} \bar{x}$
which must be the same solutions.
LS estimation is therefore the same as finding the projection onto the column vectors of $\mathbf{X}$.

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