Matrix representation of SLR stats544-1-slr Applied simple linear regression

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July 18, 2019

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Matrix representation of SLR

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Introduction

It is convenient to represent the multiple linear regression model in matrix form. For that we need:

Matrices

- Matrix operations
- Matrix representation of SLR

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Purpose of matrix representation

It is easy to set up matrices which describe the simple linear regression model. Solving this using matrix algebra gives an alternative representation of the estimators.

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Matrix form of simple linear regression

 $\mathbf{y} \in R^n$ = vector of measurements

$$\mathbf{X} = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$$

the "**X**-matrix" min $\sum (y_i - (\beta_0 + \beta_1 x_i))^2$ is equivalent to finding

$$\boldsymbol{\beta} = \left(\begin{array}{c} \beta_0\\ \beta_1 \end{array}\right)$$

to mininmize $||\mathbf{y} - \mathbf{X}\boldsymbol{\beta}||^2$ Number notation: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$

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Prediction as linear projection



Geographical representation of linear model.

Basic model: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$ where **X** is a matrix of dimensions $n \times p$ (here p = 2).

Closest prediction of \mathbf{y} within the model is via orthogonal projects of \mathbf{y} onto the plane spanned by the column vectors of \mathbf{X} .

Predicted values: $\hat{\mathbf{y}}$

The projection is in $sp{X}$, so it is a linear combination of column vectors of X so we can write

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}}$$

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Geometric solution to the simple linear regression problem

From linear algebra the matrix solution is known

$$\hat{oldsymbol{eta}} = {f X}' \dots$$

and also know
$$\hat{\beta} = \sum \dots \hat{\beta}_0 = \hat{y} - \hat{\beta}_1 \bar{x}$$

which must be the same solutions.

LS estimation is therefore the same as finding the projection onto the column vectors of X.

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