Matrix representation of SLR

(STATS544.1: Applied simple linear regression)

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Introduction

It is convenient to represent the multiple linear regression model in matrix form. For that we need:

- Matrices
- Matrix operations
- Matrix representation of SLR

Purpose of matrix representation

It is easy to set up matrices which describe the simple linear regression model. Solving this using matrix algebra gives an alternative representation of the estimators.

Matrix form of simple linear regression

$\mathbf{y} \in R^n$ = vector of measurements

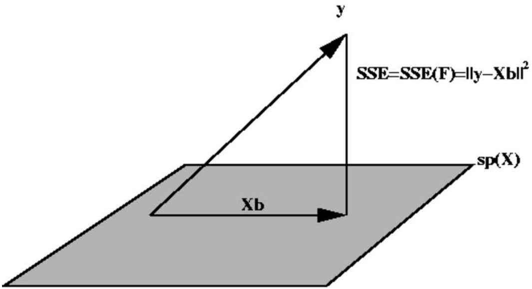
$$\mathbf{X} = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$$

the "**X**-matrix"
min $\sum(y_i - (\beta_0 + \beta_1 x_i))^2$ is equivalent to finding

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}$$

to mininmize $||\mathbf{y} - \mathbf{X}\boldsymbol{\beta}||^2$
Number notation: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$

Prediction as linear projection



Geographical representation of linear model.

Basic model: $\mathbf{y} = \mathbf{X}\beta + \mathbf{e}$ where $\mathbf{X}$ is a matrix of dimensions $n \times p$ (here $p = 2$).

Closest prediction of $\mathbf{y}$ within the model is via orthogonal projects of $\mathbf{y}$ onto the plane spanned by the column vectors of $\mathbf{X}$.

Predicted values: $\hat{\mathbf{y}}$.

The projection is in $sp\{\mathbf{X}\}$, so it is a linear combination of column vectors of $\mathbf{X}$ so we can write

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\beta}$$

for some vector, $\hat{\beta}$.

Geometric solution to the simple linear regression problem

From linear algebra the matrix solution is known

$$\hat{\boldsymbol{\beta}} = \mathbf{X}' \dots$$

and also know $\hat{\beta} = \sum \dots \; \hat{\beta}_0 = \hat{y} - \hat{\beta}_1 \bar{x}$
which must be the same solutions.
LS estimation is therefore the same as finding the projection onto the column vectors of $\mathbf{X}$.