# Group means comparisons and two-way ANOVA

(STATS546.2: Applied analysis of variance (work in progress))

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# Introduction group means comparisons

- Confidence intervals of the group means
- Contrasts
- Multiple comparisons
  - Tukey
  - Bonferroni
  - Dunnett

# Group means and confidence intervals

The estimate of the group mean $\hat{\mu}_i$ was given as $\bar{y}_{i\cdot}$, but how precise is that estimate?

Confidence interval of the mean can be constructed as:

$$\bar{y}_{i\cdot} \pm t_{1-\alpha/2,N-I}\sqrt{\frac{MSE}{n_i}}$$

# Example

# Contrasts

Confidence interval of the difference between two group means is calculated as follows:

$$\hat{d} \pm t_{1-\alpha/2,N-I} \times \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

If the confidence interval contains a zero there is not a significant difference between the two group means.

# Example

# Tukey

The Tukey test is used when there is an interest in all pairwise comparisons.

The confidence interval using method Tukeys is as follows:

$$\hat{d} \pm \frac{1}{\sqrt{2}} q_{1-\alpha,g,N-g} \times \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

where $q$ is a quantile from the studentized range distribution.

# Bonferroni

Bonferroni method of multiple comparisons is used when there is not interest in all pairwise comparisons but of some comparisons that are not based on the data.

It depends on the number of comparisons of interest how wide the confidence interval will be.

For c comparisons the Bonferroni confidence intervals become:

$$\hat{d} \pm t_{1-\frac{\alpha}{2c}, N-I} \times \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

# Dunnett

Method Dunnett's is used when there is an interest in comparing all treatment to a control or a reference group.

# Example

# Introduction to two-way ANOVA

Two-way ANOVA is used when there are two factors in the experiment,

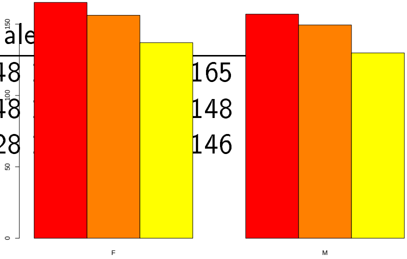
- Crop variety and fertilizer
- Age and Gender
- Price and brand

It is possible to analyze the effect of the two factors simultaneously.

There is no limit of factors that can be included in the analysis. However, the interpretations become more difficult as the number of factor increases.

# Example - Heart rate

| | Female | Male |
|---|---|---|
| age 20-30 | 160 167 157 156 158 | 148 [illegible] 165 |
| age 30-40 | 147 149 150 145 145 | 148 [illegible] 148 |
| age 40-50 | 145 157 149 157 140 | 128 [illegible] 146 |



# The two-way ANOVA model

The main effect of level $i$ of factor $A$ is

$$\alpha_i = \mu_{i\cdot} - \mu_{\cdot\cdot}$$

The main effect of level $j$ of factor $B$ is

$$\beta_j = \mu_{\cdot j} - \mu_{\cdot\cdot}$$

The main effect show how much the factor level means ($\mu_{i\cdot}$and $\mu_{\cdot j}$) deviates from the overall mean ($\mu_{\cdot\cdot}$).

The mean of each cell becomes

$$\mu_{ij} = \mu_{\cdot\cdot} + \alpha_{i\cdot} + \beta_{\cdot j}$$

and finally the ANOVA model is defined as:

$$y_{ijk} = \mu_{ij} + \epsilon_{ijk}$$

# Fitting the two-way ANOVA model

The fitting of the two-way ANOVA model is the same as in the one-way model. The method of least squares is used and the following sum is minimized.

$$S = \sum_i \sum_j \sum_k (y_{ijk} - \mu_{ij})$$

It is known that the mean of each cell ($\hat{\mu}_{ij} = \bar{y}_{ij.}$) minimizes this sum

# Example

# Advantages of two-way ANOVA

# The ANOVA table for two-way ANOVA

# Sum of squares

# Hypothesis

# F-test

# Example

# Block design

# Example

# Checking the assumptions

# Multiple comparison in the two way ANOVA

# Example

# Example