The development of a year-class

fish5102stockcatch The development of a year-class

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Background to the stock and catch equations

We will derive an equation to describe how a specified cohort develops across time.

Figure : Development of a cod cohort, as seen by the number of cod caught in groundfish surveys in Icelandic waters. Average on log-scale across several years.

Symbols

Notation Numbers at year start and end, respectively : N_0, N_1
Number at specified time, t: N_t
Short time interval: (t_1, t_2)
Length of interval: Δt
Change in stock size during interval: ΔN

Yearclass development during short time interval

Proportion which dies during a time interval =Probability of fish dying

$$\frac{\Delta N}{N} = -Z\Delta t$$

Note: Leads to power relationship: N=1000, 1000/2, 1000/4, ...

The differential equation

Differential equation:

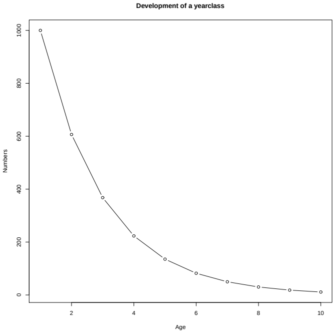
$$\frac{dN}{dt} = -ZN$$

The stock equation

Solution to differential equation:

$$N_t = e^{-Zt} N_0$$

Describes numbers at time t as compared to initial numbers.



The formula applies within a year, the figure shows yearclass development across years. Complex version:

```
N<-1000 ; N0<-N ; Z<-0.5 ; ages<-1:10
for(a in ages){
 N1<-N0*exp(-Z);
 N<-c(N,N1);
 N0<-N1;
}
plot(ages,N[1:length(ages)],type='b',
```

Development of stock

$$N_{a+1,y+1} = e^{-Z_{ay}} N_{ay}$$

Mortalities work in sequence

Suppose $Z = F + M$...

$$N_1 = e^{-Z} N_0 = e^{-(F+M)} N_0 = e^{-F-M} N_0 = e^{-F} e^{-M} N_0$$

Rates and proportions...

Logarithms or not –

$$1 - e^{-0.2} = 0.18$$

Rates and proportions
Mortality rate = 0.2 means 18% die...
Note that Z can be greater than 1!

Mortality as log-change

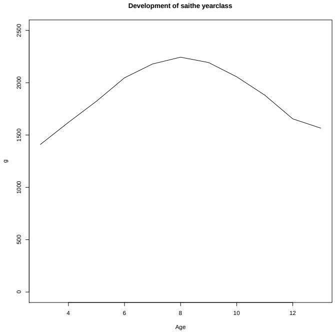
$$\ln N_{a+1,y+1} = \ln N_{ay} - Z_{ay}$$

Log scale.

Development of yearclass biomass

Given the stock equation and data on weight and mortality, it is easy to predict the development of biomass.

Notice the maximum biomass at age 8. This would be the harvest if a single cohort was raised in aquaculture.



The figure is generated using R, based on the following commands:

```
w<-c( 1.41, 1.98, 2.72, 3.73, 4.85, 6.1, 7.28, 8.34, 9.32, 10.01, 11.57  )
M<-c( 0.20, 0.20, 0.20, 0.20, 0.20, 0.20, 0.20, 0.20, 0.20, 0.20, 0.20  )
N<-1000;
N0<-N;
B<-NULL;
for(a in 1:length(w)){
 #print(a)
 N1<-N0*exp(-M[a]);
```