

fish5104vpa Assessment methods based on back-calculations

Gunnar Stefansson

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<http://mareframe-fp7.org>

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1 Simple back-calculation techniques

1.1 Backcalculating assessment methods background

Several back-calculation methods are available

For example:

- VPA (virtual population analysis): Gives (historical) stock size in numbers
- Cohort analysis and other variations: Different approximations
- "Tuning" with survey indices: Gives current abundance

1.1.1 Details

There are a wide number of assessment methods in use around the world. These can be classified according to their statistical nature, their assumptions, their use of various available data sets and so on.

The grandfather of all assessment methods is the Virtual Population Analysis (VPA). This method will be described in some detail, following an introduction on the basic principles of back-calculation.

1.1.2 Examples

Example 1.1. Throughout this section, data on cod in Icelandic waters will be used as an example to illustrate stock assessment methodology.

1.2 Exceedingly simple stock assessments

VPA = Virtual Population Analysis

Start with simplified VPA

- no differential equations
- fishing not concurrent with natural mortality

Assumptions:

- 18% natural mortality
- Entire cohort caught in the last year

1.2.1 Details

In order to illustrate the principles of VPA, we start with a back-calculation method which is considerably simpler. Such back-calculations work by looking at a single cohort and estimating its size backwards in time.

Suppose that the fishery each year is operated at the beginning of the year, before any natural mortality takes place.

1.2.2 Examples

Example 1.2. Consider cod in Icelandic waters where it is commonly assumed that natural mortality is 18% per year.

In order to start the back-calculations, some assumptions on the last year are needed. The simplest possible assumption is to assume that the entire cohort has been fished out at the end of the year.

1.3 Simple back-calculation: one year

1987 = previous year

- catch = 46 thousand fish
- stock (cohort) size before catch in last year = catch = 46 thousand fish
- stock (cohort) size at beginning of year = stock (cohort) size before catch \div 0.82
- $N = 46 \div 0.82 = 56$ thousand fish

1.3.1 Details

Simplified back-calculations start by assuming that the year-class was fished during the last year and that natural mortality occurs before fishing this year. Thus, immediately before fishing incurs, the year-class is about to be fished and this is the only source of mortality.

Therefore the stock just before fishing is the same as the amount caught. Given the proportion, p , that dies due to natural causes (before fishing), it is clear that one must have $1 - p$ as the proportion that survived natural mortality (and thus got caught) and therefore division by $1 - p$ gives the number of fish at the start of the ultimate year.

With this simple technique one obtains the numbers at the end of the penultimate year. Using the corresponding assumptions for that year, one can add the catches and divide by the survivorship to obtain the numbers at the beginning of the penultimate year.

1.3.2 Examples

Example 1.3. Simple back-calculations for a cohort of cod in Icelandic waters.

1987: catch = 46 thousand fish

stock (cohort) size before catch last year = catch = 46 thousand fish

stock (cohort) size at beginning of year = stock (cohort) size before catch \div 0.82 = $46 \div 0.82 = 56$ thousand fish

1986: catch = 103 thousand fish

stock (cohort) size before catch = $56 + 103 = 159$ thousand

stock (cohort) size at beginning of year = $159 \div 0.82 = 194$ thousand fish

1985: catch = 217 thousand fish
 stock (cohort) size before catch = $194 + 217 = 411$ thousand fish
 stock (cohort) size at beginning of year = $411 \div 0.82 = 501$ thousand

1.4 Simple backcalculation - full cohort

Year	Age	Catch	p	Cohort size
1987	13	46	100	56
1986	12	103	64	194
1985	11	217	52	501
1984	10	512	50	1235
1983	9	2054	62	4011
1982	8	7666	65	14241
1981	7	12710	47	32867
1980	6	15119	31	58519
1979	5	13772	19	88160
1978	4	16286	15	127375
1977	3	2614	2	158523

1.4.1 Details

The simple back-calculation procedure can be extended backwards through the life span of the year-class.

When results from this simple method are compared to results from more highly developed techniques it turns out that the differences in historical stock sizes are usually minimal. It is therefore possible to get a decent historical overview of stock development using very simple numerical methods.

1.4.2 Examples

Example 1.4. Simple back-calculations for a cohort of cod in Icelandic waters.

The results from these computations can be summarized in a single table, which describes the size of the cohort in time, starting at age 13 and going backwards to the start of the fishery.

Year	Age	Catch	p	Cohort size
1987	13	46	100	56
1986	12	103	64	194
1985	11	217	52	501
1984	10	512	50	1235
1983	9	2054	62	4011
1982	8	7666	65	14241
1981	7	12710	47	32867
1980	6	15119	31	58519
1979	5	13772	19	88160
1978	4	16286	15	127375
1977	3	2614	2	158523

1.5 Simple backcalculation - revised assumptions

- Revise the assumption of catching everything in the last year
- More reasonable to assume 50%

1.5.1 Details

When considering the back-calculated proportion caught, it tends to be obvious that the initial assumption of everything being caught is incorrect. This assumption can easily be modified.

1.5.2 Examples

Example 1.5. Simple back-calculations for a cohort of cod in Icelandic waters under other assumptions.

The assumption of catching the entire cohort in the last year can easily be remedied and different assumptions can be tested. For example, if it is instead assumed that the final catch is only 50% of the cohort, then the following table is obtained.

Year	Age	Catch	p	Cohort size
1987	13	46	50	112
1986	12	103	47	262
1985	11	217	45	584
1984	10	512	46	1337
1983	9	2054	60	4135
1982	8	7666	64	14392
1981	7	12710	46	33051
1980	6	15119	31	58744
1979	5	13772	18	88435
1978	4	16286	15	127710
1977	3	2614	2	158931

Although this simple approach is quite illustrative, it has some serious shortcomings. In particular, natural mortality and fishing mortality are assumed to occur at separate times of year and this needs to be alleviated.

2 Virtual Population Analysis (VPA)

2.1 Traditional VPA

Assumptions of VPA:

- F is assumed to be known in the last year
- M is assumed to be known
- Catch is assumed to be without error

VPA is based on the following equations:

Stock equation

$$N_{a+1,y+1} = e^{-Z_{ay}} N_{ay}$$

Catch equation

$$C_{ay} = \frac{F_{ay}}{Z_{ay}} (1 - e^{-Z_{ay}}) N_{ay}$$

2.1.1 Details

The virtual population analysis starts with the basic stock and catch equations.

Definition 2.1. Stock equation:

$$N_{a+1,y+1} = e^{-Z_{ay}} N_{ay}$$

Definition 2.2. Catch equation:

$$C_{ay} = \frac{F_{ay}}{Z_{ay}} (1 - e^{-Z_{ay}}) N_{ay}$$

The catch at age in numbers is known and will be assumed to be measured without error. It will also be assumed that natural mortality is known.

The classical virtual population analysis, (Gulland 1965), is based on the assumption that the fishing mortality rates of the last year are known and can be inserted as assumed values. Further, values have to be assumed for the fishing mortality rates for the oldest fish. In what follows, these assumptions will be taken for granted but in later sections it will be shown how it is possible to estimate the mortality rate for the oldest fish and the last year.

2.1.2 Examples

Example 2.1. If Icelandic cod are taken as an example, the following table contains an estimate of the number of fish by age in the catches during the years 1974-1979:

<http://tutor-web.net/fish/fish5104vpa/lecture20/cod-catches-by-age.dat>

Example 2.2. An estimate of natural mortality for cod in Icelandic waters indicates that M may be close to 0.2 for three year old cod and older. This number will be used in what follows. Of course the assumptions behind these numbers need to be checked and the effect of deviations from such assumptions needs to be evaluated.

2.2 Traditional VPA - The last year

F is given in the last year

M is given

Catch is given

Solve the catch equation with respect to N

$$N_{ay} = \frac{C_{ay}}{\frac{F_{ay}}{Z_{ay}}(1 - e^{-Z_{ay}})}$$

2.2.1 Details

For the last year the catch equation can be used to compute the stock size at the beginning of the last year since in that equation all is known except N_{ay} and this can be computed.

Definition 2.3. Stock size equation:

$$N_{ay} = \frac{C_{ay}}{\frac{F_{ay}}{Z_{ay}}(1 - e^{-Z_{ay}})}$$

It follows that one can deduce knowledge of stock size from the assumption on fishing mortality along with the assumption that the catch is known without error.

2.2.2 Examples

Example 2.3. In the cod example it is possible to start by assuming that the total mortality is about 0.8 for the oldest fish and the fishing mortality is therefore 0.6. It should be mentioned that it is better to start with values that are too high rather than small since there is some danger of the method "getting stuck" at consistently too-low

values, if the initial values are too low. Having fixed the fishing mortality rate for the oldest fish, e.g. from catch-curve analysis, one needs to decide whether this should be the same across all age groups or whether there should be some change in F at younger ages.

It will become clear that the fishing mortality is considerably lower for the younger fish and therefore it is possible, for example, to assume that the fishing mortality by age in the last year can be set up to start at low values and subsequently increase towards some plateau.

In this example it is simply assumed here that the mortality due to fishing is reduced linearly for fish younger than 8 years old, and that the fishing mortality rate is zero at age two. This assumption is somewhat "out of the blue", but it will be seen later that the assumption can be revisited after a first round of computations is complete.

Age	3	4	5	6	7	8	9	10	11	12	13	14
F1979	0.1	0.2	0.3	0.4	0.5	0.6	0.6	0.6	0.6	0.6	0.6	0.6

Example 2.4. To continue with the cod the stock size in numbers for the last year (1979) is computed using simple division as seen in the following table:

<http://tutor-web.net/fish/fish5104vpa/lecture20/1979codstock.dat>

Here, the number $\psi = \frac{F_{ay}}{Z_{ay}}(1 - e^{-Z_{ay}})$ is what is divided into the stock size equation, and this is the proportion which is caught from the stock.

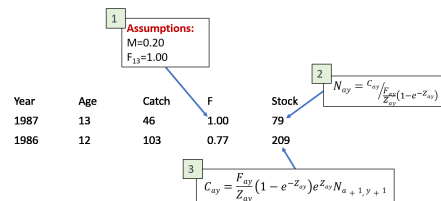
2.3 Traditional VPA - Penultimate year

Have N for last year ($y + 1$)

$$C_{ay} = \frac{F_{ay}}{Z_{ay}}(1 - e^{-Z_{ay}})e^{Z_{ay}}N_{a+1,y+1}$$

The catch equation is not easy to solve with regards to F .

Use numerical methods



2.3.1 Details

For the second last year the catch is known along with the stock size in the following year. By combining the stock and catch equations it can be seen that the catch in the second last year is connected to the stock size in the year that is following through the equation:

$$C_{ay} = \frac{F_{ay}}{Z_{ay}}(1 - e^{-Z_{ay}})e^{Z_{ay}}N_{a+1,y+1}$$

In this equation all quantities are known except F_{ay} , which can therefore, in principle, be solved from the equation. This equation is, however, non-linear in the fishing mortality F_{ay} and a direct analytical solution can not be obtained.

It is of course possible to solve the equation by testing different values of the fishing mortality coefficient. Earliest approaches were to tabulate the proportion $\psi = \frac{F}{F+M}(1 - e^{-(F+M)})e^{F+N}$ for a variety of F and M -values and use these to look up which F corresponded to the observed fraction $C_{ay}/N_{a+1,y+1}$.

The preferred current approach is to use established numerical methods, implemented either as popular solver functions in spreadsheets or minimizers in other packages such as R. In this fashion the fishing mortality in the last but one year can be estimated for age groups other than the oldest one.

2.3.2 Examples

Example 2.5. This method of inverting the catch equation can be used in the cod example to obtain the following mortality rates in the second last year:

<http://tutor-web.net/fish/fish5104vpa/lecture20/1978codf.dat>

2.4 F on the oldest age

F of the oldest ages is an average of younger years

2.4.1 Details

At this point in time the fishing mortality has been computed for the last but one year, except for the oldest fish. As mentioned above, the basic virtual population analysis assumes that this mortality coefficient is given.

Depending on the stock, it may be reasonable to assume that fishing mortality of the oldest fish is the same as some of the slightly younger ages. Thus, the price of "large" fish may be similar, regardless of whether these fish are 5kg or 7kg, etc. This approach is one possible method to estimate the fishing mortality for the oldest fish and therefore those numbers do not need to be assumed as in the original VPA.

2.4.2 Examples

Example 2.6. For the cod off Iceland it is natural to assume that the fishing mortality for the oldest fish is the same as the fishing mortality for the oldest but one age group. Another reasonable assumption is that the fleet does not make much of a difference between the oldest age groups. It is therefore possible to take the average, for example of the mortality rate of 10-13 year old fish, and assume that the average is valid for the oldest age group. This average turns out to be 0.538 which can be used as an F on 14 year old fish in the year 1978.

Example 2.7. The cod stock above was estimated by assuming that the fishing mortality for the oldest fish in the last year was about 0.6, after which deviations from this assumption were investigated. From the figure it would seem that the mortality rate had reduced during this time period at least during the years 1975-1977. In this light it may be natural to assume that the effort in 1979 was about the same as in 1978 rather than using more years in average. Thus, it might be natural to expect the average F for ages 5-10 (here denoted \bar{F}_y) to have been about 0.5 in the year 1979, although it is certainly not possible to exclude other possibilities if these data alone are the basis for the analysis.

2.5 Backcalculating a cohort

Can now back-calculate the entire cohort, year by year

2.5.1 Details

It is now possible to use exactly the same method to estimate the stock size of the second last year and then continue to back-calculate stock size and mortality rates back in time. By using the back-calculations it is possible to estimate historical stock sizes in a relatively simple fashion. The basic drawback to this method is of course the prerequisite that the mortality rate of the last year has to be known.

2.5.2 Examples

Example 2.8. This preliminary assessment of the cod stock can be completed by finishing the above computation based on the assumption which was mentioned, i.e. to use a very rough estimate of the mortality rates in the last year and the average over ages 10- 13 as an estimate of the F on age 14 each year in the back- calculations.

F backcalculation:

<http://tutor-web.net/fish/fish5104vpa/lecture20/fbackcalculation.dat>

N backcalculation:

<http://tutor-web.net/fish/fish5104vpa/lecture20/nbackcalculation.dat>

2.6 VPAs using different input F s should be compared

2.6.1 Details

It so happens, that if fishing mortality rates are high then the assumption on the mortality in the last year has little effect on the conclusion for the former years, but this needs to be verified on a case-by-case basis.

Thus, conclusions for the years before 1988 will be almost independent of the input for the year 1991 if fishing mortality rates are in the range 0.8-1.0 and natural mortality is about 0.2. The reason for this is that if F is high, then there is not much left of year-classes, for example at the age of 9, and assumptions on how much is left do not have much effect on the estimate of year-class strength at age 3.

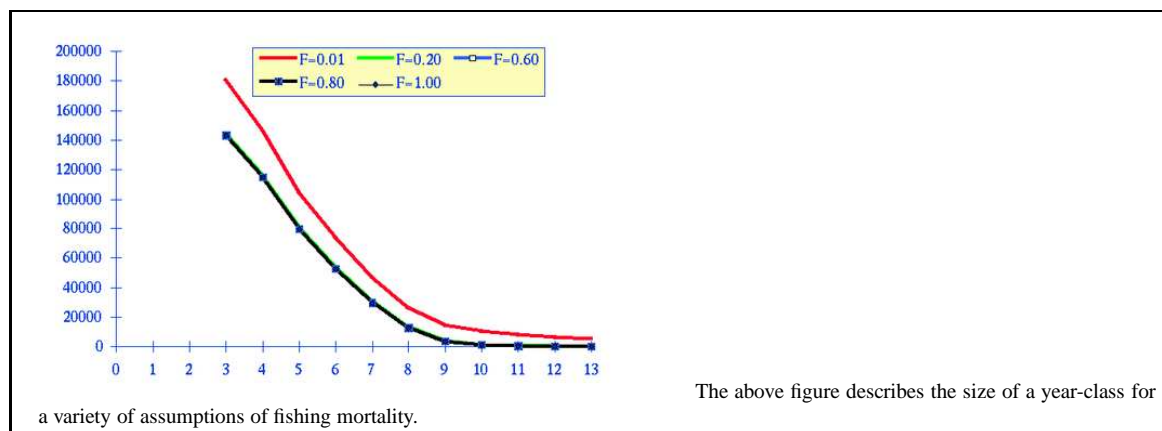
2.6.2 Examples

Example 2.9. To continue with the cod example, the year-class strength can be estimated based on a different assumption on the fishing mortality in the last year. The average of the fishing mortality rates for the ages 5-10 was taken to be 0.5 in the previous table.

Now different assumptions can be tested, for example to multiply the mortality rate in 1979 in such a fashion that this average becomes 0.3 and 0.8. If this is done, the following table on fishing mortality rates appear:

<http://tutor-web.net/fish/fish5104vpa/lecture20/lowhighf.dat>

2.7 Convergence properties of VPA



2.7.1 Details

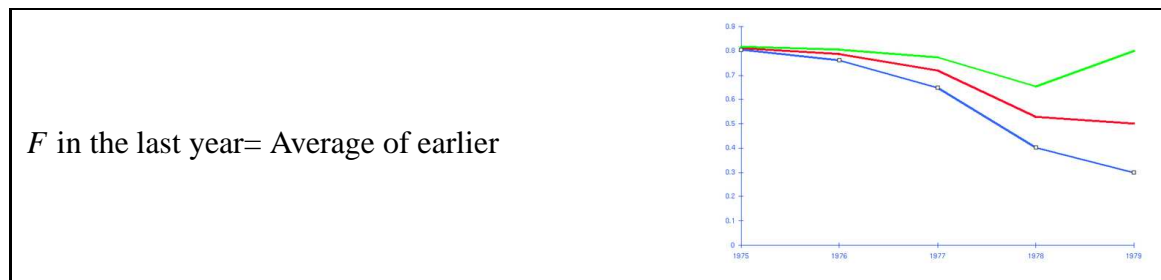
Virtual Population Analysis has an important feature, namely backwards convergence. This effect, seen in all computed quantities, basically implies that when back-calculations are used, historical values tend to converge to numbers which are the same, regardless of what assumptions are used for the last year. In particular, this implies that VPA can be used to obtain historical year-class sizes. This also implies that accurate information can be obtained about historical fishing mortality, which is crucial when using VPA.

The convergence property is a function of the level of total mortality; if total mortality is high, then convergence is fast.

2.7.2 Examples

Example 2.10. The accompanying figure describes quite well the convergence properties of VPA. Here it is shown how independent the historical results for stock size are on the initial value of fishing mortality. Only for extremely low values of F is it possible to force the trajectory away from about 140 million recruits.

2.8 Revising assumptions for the last year



2.8.1 Details

In the absence of other information, it is important to test a variety of different assumptions on the "input" fishing mortality.

It is common practice to verify how far back the effects of the assumptions go.

The speed of the convergence depends on several quantities, the most important being the true level of fishing mortality. If the actual historical fishing mortality has been low, then convergence will be poor and VPA will not be a good assessment method.

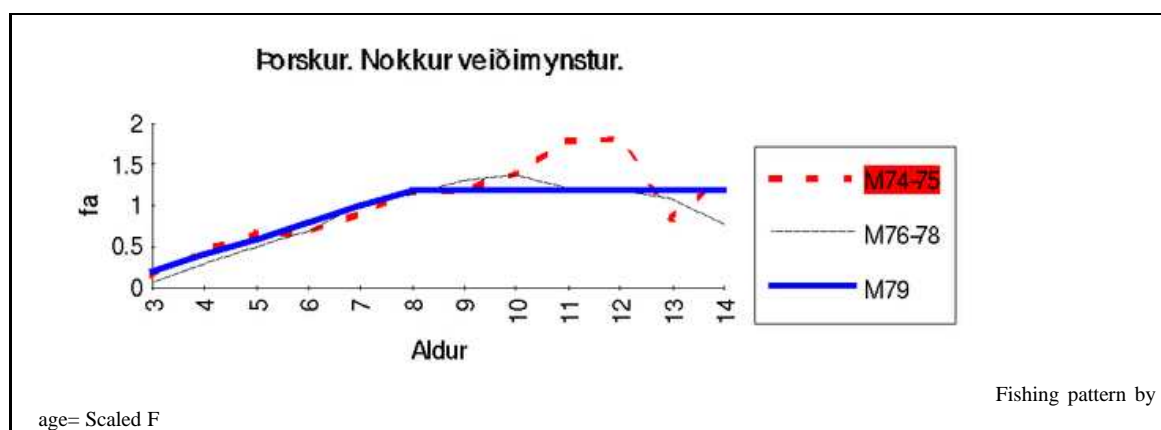
In order to summarize the information about the effect of fishing, an average \bar{F}_y over a few age groups is often used, and this is considered a time series.

Having looked at the trend in \bar{F}_y , it is natural to consider how the assumption made in the last year appears to fit with the back-calculations. In the absence of any other information, there would not seem to be reason to use an assumption which implies a jump in fishing mortality in the last year. For this reason the assumption for the last year is commonly revised to reflect an assumption of constant fishing mortality over the last years of the assessment.

2.8.2 Examples

Example 2.11. In the Iceland cod case study it is seen that considerably different input fishing mortality is only reflected backwards in time for some 2 years prior to the most recent.

2.9 The selection pattern



2.9.1 Details

The average fishing mortality measure per year, \bar{F}_y , is an overall measure of the probability that a certain fish gets caught.

The fleet as whole may be homogeneous, i.e. the composition of the fleet and the use of fishing gear does not change much in time. In this case it may be reasonable to assume that the proportional effort in regards particular age groups does not change much from one year to the next. Under such circumstances a separable model should apply.

Definition 2.4. Fishing mortality for a homogeneous fleet:

$$F_{ay} = \bar{F}_y s_a$$

s_a =fishing pattern or selection pattern

In this model s_a is defined in such a fashion that its average is one on age groups used to calculate the average \bar{F}_y .

In reality a fleet is usually composed of various groups of vessels or sub-fleets, such as gillnets and trawlers using bottom trawl etc. Usually the fleet composition changes considerably over time. For example, in a quota system quotas will move from one vessel group to another and thus the fishing pattern for the fleet as a whole will change even though the fishing pattern for the individual fleets does not change. Furthermore, changes in fishing gear, such as changes in mesh size, will affect the fishing pattern of the individual vessel groups.

2.9.2 Examples

Example 2.12. To complete the preliminary VPA for cod in Icelandic waters, it is now necessary to investigate the probable mortality rate for the younger fish since the assumption is simply that it would reduce linearly from that of the older fish.

In the above picture the selection pattern used in 1979 is shown along with two average selection curves, the average over the years 1976-1977 and 1974-1975. This selection pattern as defined above is simply computed by dividing \bar{F}_y into all the mortality rates in the year, given a new column with the selection pattern for each year. The average of a few years of these patterns indicates how the effort by the fleet is proportionally split between the age groups. From the figure it seems clear that the selection pattern which was chosen was not at all absurd, although it may possibly be improved upon by lowering the fishing mortality for the youngest fish, particularly the three year old.

2.10 Revising the last year

To re-estimate the last year:

- assume constant F
- use effort data
- use the survey index
- use more elaborate "tuning"
- move to statistical methods

2.10.1 Details

Given the potential of VPA or other back-calculation methods to provide the historical stock estimate, information can clearly be obtained on the selection pattern (s_a) and the trend in the fishing mortality coefficient (\bar{F}_y). If convergence is good, then these are well-established except for the last few years.

If it is known that the effort has been constant for some years, then one could simply assume that the fishing mortality coefficient in the last year is the average of the fishing mortality coefficient in some previous years. Thus, it is simple to obtain a decent estimate of mortality rates by doing virtual population analyses with high fishing mortality rates and then use some average of previous years as input for a new analysis.

This would most appropriately be used to re-estimate the overall (i.e. average across ages) fishing mortality rate, \bar{F}_y , for the last year. If the fleet composition can be assumed to not have changed for some time, then a historical selection pattern can also be assumed. The combination of the overall level and the fishing pattern now gives a revised vector of fishing mortalities for the last (terminal) year.

If it is known that effort has increased, for example by 20% from the average of the years 1980-1985 to 1991, the average of the mortality rate of those years can be multiplied by 1.2 to obtain a new input for the virtual population analyses.

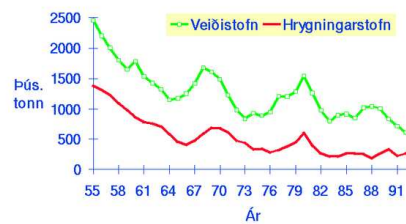
Other, much better, methods are usually applied to tune the stock estimate according to information from a survey or information about the effort supplied by individual parts of the fleet. These methods are usually called "tuning" methods.

Note 2.1. Tuning methods are methods designed to to "tune" the terminal fishing mortality.

Much better than these, tuning methods are methods which take into account the statistical properties of the data. These statistical assessment methods are much more elaborate and time-consuming so there is always a case to be made for the simpler VPA-style methods, which can be used to quickly get a rough feel for the level of exploitation of the stock. The more elaborate methods would be expected to provide a better basis for providing advice or understanding of the stock dynamics.

2.11 I-cod, Historical stock development

- Assume M
- Assume F in last year
- Back-calculate
- Check consistency: Fishing pattern trend
- Revise F in last year
- Repeat VPA



2.11.1 Details

It has been seen in the previous sections how it is possible to use fairly simple methods of back-calculation to obtain estimates of historical stock size. It is also clear that the estimates of the most recent years are highly uncertain.

3 Cohort analysis

3.1 Other approximations - Cohort analysis

N assumed known in the last year
 M assumed known and inflicted mid-year
Catch known
Revised equation easily solved with regards to N

3.1.1 Details

Several variations exist on VPA. A simple variation is, for example, to create a model whereby it is assumed that a given proportion of an age group dies before fishing commences and where the catch is also a specific ratio. The model that will be described here, however, assumes that fishing takes place at around the middle of the year and that natural mortality will affect the stock before and after the fishing season. Natural mortality is usually taken to be constant in age and time, but it would also be possible to write the same equations using variable natural mortality.

The VPA equations assume that fishing and natural mortality rates are constant within the basic time period considered (usually a year). Naturally, this is an unrealistic assumption as fishing rates will almost always vary within the year. Notably for many species there are separate spawning seasons, others migrate at certain times of year into the fishing area and so forth.

It will be seen that the exact assumption used regarding the timing of the fishery (mid-year vs evenly over the year) has little effect on the final outcome but assuming a mid-year fishery is considerably simpler than the VPA approach. For this reason many contemporary estimation procedures tend to use the cohort equations internally, particularly when back-calculation is used.

3.1.2 Examples

Example 3.1. If data on catches in numbers at age are placed in a file called "nsplaice.canum", with years down and ages across, they can be read into R using the following command sequence (these data are available at: <https://notendur.hi.is/gunnar/tutor-web.data/biol/nsplaice.canum>)

```
cnum<-read.table("https://notendur.hi.is/~gunnar/tutor-web.data/biol/
/nsplaice.canum",header=T,as.is=T,check.names=F)
cnum<-as.matrix(cnum)
```

where the options to read.table ensure that the column names are not modified by R and the second line converts the data frame to a matrix.

Since the rows and columns have names, these data can be indexed in either of two ways:

```
# Example: Look at 2 year olds in 1991
print(cnum[22,1])
# Note that the columns and lines have names so this is the same as
C2<-cnum["1991","2"]
print(C2)
```

3.1.3 Handout

The following data on North Sea plaice (obtained from ICES, <http://www.ices.dk>) is a typical set of catches in numbers at age:

<http://tutor-web.net/fish/fish5104vpa/lecture30/northseaplaice.dat>

Note that the number of age groups shown is less than commonly used or made available for this plaice stock and that the data is deliberately set up with ages across and years down to make it easier to add a new year when new data become available. This is a bit contrary to the notation normally used, i.e. the data are usually denoted C_{ay} which implicitly assumes that the years are across and the ages down.

The years and ages are labelled. This helps considerably in maintaining the data file and also in labeling data frames or matrices when the data are read into a statistical package.

3.2 Cohort analysis - mid-year formula

First project the cohort to mid-year assuming no catches

Then subtract mid-year catches so cohort size after catches becomes:

$$N_{ay}e^{-M/2} - C_{ay}$$

End of year cohort size becomes:

$$N_{a+1,y+1} = (N_{ay}e^{-M/2} - C_{ay})e^{-M/2}$$

3.2.1 Details

A given age group, a , is of the size N_{ay} at the beginning of the year, y . Provided a period of six months passes without fishing, the size of the year-class at the middle of the year can be calculated.

Definition 3.1. Mid-year stock size without fishing:

$$N_{ay}e^{-M/2}$$

If the entire catch is taken at the mid-year mark and the year-class is reduced to a population size estimated from the end-of-year equation.

Definition 3.2. End-of-year population size with mid-year fishing:

$$N_{ay}e^{-M/2} - C_{ay}$$

Then the year-class decreases due to natural mortality.

Definition 3.3. End-of-year population size with mid-year fishing and natural mortality:

$$N_{a+1,y+1} = (N_{ay}e^{-M/2} - C_{ay})e^{-M/2}$$

3.2.2 Examples

Example 3.2. Consider the full plaice data matrix as presented in the handout. Forward predictions of stock size using R can be done as follows. First extract data from the catch matrix and then investigate parts of the prediction formula.

<http://tutor-web.net/fish/fish5104vpa/lecture30/stocksizepredictions.r>

3.3 Cohort analysis, back-calculations

$$N_{ay} = (N_{a+1,y+1}e^{M/2} + C_{ay})e^{M/2}$$

3.3.1 Details

Rather than projecting the cohort numbers forward in time, it is possible to back-calculate the numbers using the exact same assumptions. Reversing the earlier forward projection yields the following formula:

$$N_{ay} = (N_{a+1,y+1}e^{M/2} + C_{ay})e^{M/2} \quad (1)$$

This equation enables simple back calculations of stock sizes. For a given stock size, the last year can be back calculated by using the above equation and thus complete the stock size chart.

It should be noted that using the above equation gives almost the same results as the VPA equations. This means, among other things, that the results of stock size estimations with the VP-Analysis are relatively independent of the assumption that there is constant fishing throughout the year.

3.3.2 Examples

Example 3.3. Suppose we assume there is a single fish left as an 11-year old at the end of 1995 (i.e. 12-year old at the beginning of 1996).

```
M<-0.15
N12<-1
C11<-cnum["1995", "11"]
# Then we can back-calculate the numbers
N11<-(N12*exp(M/2)+C11)*exp(M/2)
print(N11)
```

3.4 Back-calculating a full year

Fix year, y and back-calculate

$$N_{ay} = (N_{a+1,y+1}e^{M/2} + C_{ay})e^{M/2}$$

for each age. Get all but oldest.

3.4.1 Details

For a fixed year, y , one can back-calculate the stock in numbers at age for each age.

Definition 3.4. Single year back-calculation:

$$N_{ay} = (N_{a+1,y+1}e^{M/2} + C_{ay})e^{M/2}$$

Denote the oldest age by A . Given initial numbers for ages $2, \dots, A$, the formula will give numbers at age in the previous year, for ages $1, \dots, A - 1$.

3.4.2 Examples

Example 3.4. One can also take all the data for a year (e.g. 1995) and back-calculate an entire year. Note that we need to assume something for 1996. The assumption here is simply gibberish.

```
C95<-cnum["1995",]
print(C95)
N96<-c(60000,60000,60000,40000,40000,40000,10000,10000,10000,1)
names(N96)<-as.character(3:12) # Note these are beg. yr ages 3
    through 12
N95<-(N96*exp(M/2)+C95)*exp(M/2)
names(N95)<-as.character(2:11) # Note the shift
```

The catch data contains ages 2-11 and the assumption was made for age 12.

```
print(C95)
print(N96)
print(round(N95))
```

We are now missing age 12 in the stock for 1995 - need to add that before we set up the full table.

3.5 Making ends meet



3.5.1 Details

Consider backcalculating from year 1975 to year 1974 and suppose the ages in the catch are ages 2-10. During a back-calculation only ages from age 3 (second age group) through the oldest-plus-one (11) are used for year 1975. After back-calculation numbers have been generated for ages 2-10 in year 1974, i.e. for all true ages, but not for the oldest-plus-one, which is needed to continue the back-calculation.

When this approach is taken, it is customary to assume values for the numbers at age in the oldest age group plus one each year and for the last data year plus one.

3.5.2 Examples

Example 3.5. Consider the plaice example again, where we now plan to back-calculate to the beginning of the last two data years based on assumed values after the last data

year and after the oldest age each year. This could be done in R as follows:

<http://tutor-web.net/fish/fish5104vpa/lecture30/2yearbackcalculation.r>

3.5.3 Handout

A more complete algorithm can be set up as a function in R.

In this case one wants to take the catch matrix as an argument and automatically use the dimensions of the matrix to find the number of year and ages.

Note that in this example, the number of fish in the last-plus-one year and last-plus-one age are also taken as arguments as is natural mortality.

```
# cohortN: Cohort analysis based on assumed Noldest and Nlast
# cnum: Y*A matrix of catch-in-numbers
# Nlast: length A vector of numbers in the last year
# Noldest: length y+1 vector of numbers of age A+1
cohortN<-function(cnum,M,Nlast,Noldest){
  A<-ncol(cnum)
  Y<-nrow(cnum)
  Nmat<-c(Nlast,Noldest[Y+1]) # Set up the last line of the full N-
    matrix
  N1<-Nlast # Ages 1 through oldest
  for(y in Y:1){ # Loop over years in the catch matrix
    Cline<-cnum[y,] # A line in the catch matrix
    Ntmp<-c(N1[2:A],Noldest[y+1]) # Ages 2 through oldest plus one
    NO<-(Ntmp*exp(M/2)+Cline)*exp(M/2) # Start of year, ages 1:A
    NO<-c(NO,Noldest[y]) # Stock numbers, ages 1:(A+1) at start of
      year
    Nmat<-rbind(NO,Nmat)
    N1<-NO # Prepare to repeat the loop
  }
  return(Nmat)
}
```

If the above function definition is stored in an R command file called "cohortN.r", then the following describes an R session which first defines the function by reading in the command file and then uses the function.

```
source("http://tutor-web.net/fish/fish5104vpa/lecture30/cohortN.r")
cnum<-read.table("http://notendur.hi.is/~gunnar/tutor-web.data/biol/
  nsplaiice.canum", header=T,as.is=T,check.names=F)
cnum<-as.matrix(cnum)
cnum
Nmat<-cohortN(cnum,0.15,1:ncol(cnum),1:(nrow(cnum)+1))
Nmat
```

In this example, the starting values for stock size were simply the integers 1:5 and 1:4 for the last year and age, respectively. Naturally, these are not very sensible population

numbers. It would make much more sense to at least have the initial values of the same order of magnitude as the catches through the following command sequence:

```
source("http://tutor-web.net/fish/fish5104vpa/lecture30/cohortN.r")
cnum<-read.table("http://notendur.hi.is/~gunnar/tutor-web.data/biol/
  nsplaice.canum", header=T, as.is=T, check.names=F)
cnum<-as.matrix(cnum)
A<-ncol(cnum)
Y<-nrow(cnum)
Noldest<-mean(cnum[,A]) # Assume we fish/leave half the oldest
Noldest<-rep(Noldest, Y+1)
Nlast<-apply(cnum, 2, mean) # Similar for the last+1 year
Nmat<-cohortN(cnum, 0.15, Nlast, Noldest)
```

In the last case, the stock-numbers table becomes the following:

Nmat

and it is worth noting, that the table appears much more sensible than one where the last row and column are all filled with "1". It is also worth noting that the values increase in each column when looking upwards (except the last column) and this may indicate that the assumptions in the last year are somewhat too low.

3.6 Cohort analysis - F-computations

Compute F within table afterwards:

$$F = \ln(N_0/N_1) - M$$

or

$$F_{ay} = \ln(N_{ay}/N_{a+1,y+1}) - M_{ay}$$

3.6.1 Details

By its very nature, backwards computations using cohort analysis do not use fishing mortalities. Indeed, since it is assumed that catches are taken instantaneously mid-year, one might argue that instantaneous fishing mortality rates are zero except at the moment when the catches are taken, at which time the fishing mortality rates are infinite.

However, acknowledging that the method is an approximation and that the concept of fishing mortality rates is useful, it is certainly possible to compute a fishing mortality rate from the log-ratio of adjacent stock sizes, subtracting natural mortality in an appropriate manner:

$$F_{ay} = \ln\left(\frac{N_{ay}}{N_{a+1,y+1}}\right) - M_{ay}.$$

This results in a matrix of F -values. The size of the matrix is one less column and row than the N -matrix.

If the matrix of stock numbers had one more year and age than the catch-matrix, then the matrix of fishing mortalities will be the same size as the catch matrix. If the N -matrix is the same size as the C -matrix, then the F -matrix is missing the last year and oldest age of the C -matrix. In either case, it is clear that the last year and oldest age will not be useful

since any numbers for these will completely depend on the assumption on stock numbers in or after the last year and age.

A method is therefore needed to compute the fishing mortality on the oldest fish and in the last year.

3.6.2 Examples

Example 3.6. Easy in principle - careful with indices!

```
log(Nmat[2,3]/Nmat[3,4]) - M # Simple example
```

Assume a fixed size of the N-matrix:

```
log(Nmat[1:10,1:2]/Nmat[2:11,2:3]) - M # The whole matrix
```

Finally, we can extract the size of the matrix in order to obtain a perfectly general method for computing the fishing mortality.

```
A <- ncol(Nmat); Y <- nrow(Nmat)
```

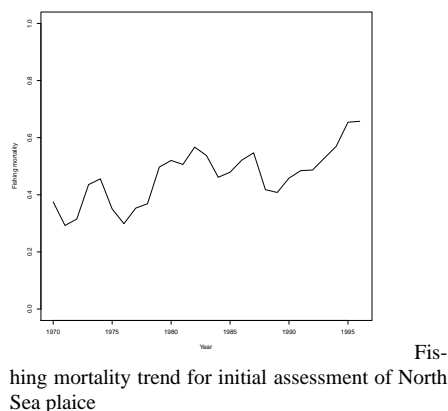
```
Fmat <- log(Nmat[1:(Y-1),1:(A-1)]/Nmat[2:A,2:Y]) - M # The general case
```

3.7 The trend in fishing mortality

Want an overall measure of fishing mortality per year.

Use average, \bar{F}_y , over selected age group.

Common options: Oldest ages or well-sampled ages.



3.7.1 Examples

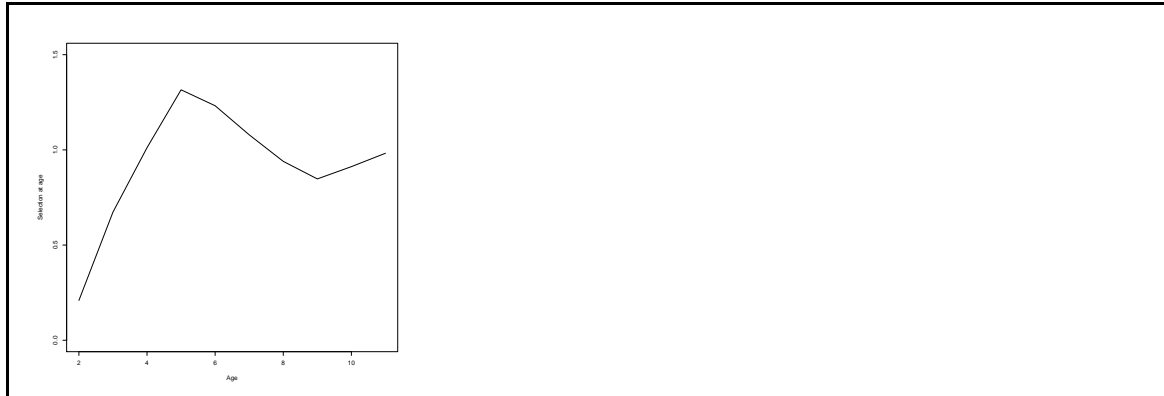
Example 3.7. Computing the trend in fishing mortality, using a fixed set of ages and years can be done in R using fairly simple matrix algebra. Suppose the stock in numbers by age and years has been computed and placed in the matrix Nmat. Then one can first compute the fishing mortality and subsequently take the main across a range of ages within each year, to compute the trend.

```
Fmat <- log(Nmat[1:10,1:6]/Nmat[2:11,2:7]) - M # The fishing mortality table except for oldest and last
```

```
Fbary <- apply(Fmat[,3:5],1,mean) # Compute the average annual fishing mortality
```


For the North Sea plaice it is seen that the trend is fairly smooth, but appears to be increasing quite strongly in the most recent years. In the present setting, this is an artifact of the assumed stock size in numbers in the last year. In the absence of other information, it would be more natural to assume that the fishing mortality in the last year was somewhere close to the average of earlier years.

3.8 The selection pattern



3.8.1 Details

If the fishing mortality is F_{ay} with an average fishing mortality for each year, \bar{F}_y , given as the average of F_{ay} across some selected age groups, then the annual selection pattern is defined by the annual selection pattern equation.

Definition 3.5. Annual selection pattern equation:

$$s_{ay} = F_{ay} / \bar{F}_y$$

Basically, the approach existence of an underlying long-term average selection pattern, of which the annual selection pattern is a realization.

The average selection pattern, s_a , is now estimated as the average across a selected range of years of s_{ay} within each age group.

Usually the selection pattern for a stock is assumed to increase steadily towards a maximum. This is not the only possibility, however, and the actual shape must be investigated on a case-by-case basis.

3.8.2 Examples

Example 3.8. Although the computation appears trivial, there are a few issues to be taken care of. Firstly, when this is computed in R, one must note how matrices and vectors operate together. Basically, if a matrix is divided by a vector, then the first element of the vector is divided into the first line of the matrix and so forth, exactly in the manner needed in this example:

```
matrix(1:16,ncol=4)
matrix(1:16,ncol=4)/c(1,2,3,4)
```

It follows that the matrix of annual selection patterns is obtained by simple division in R

```
selpat<-Fmat/Fbary
```

and the average pattern can be obtained by taking an average over a reasonably chosen range of years. The choice of range depends on the situation at hand. For example, one attempts to use a fairly recent year range in order to avoid a period involving a gear change but long enough to obtain a smooth pattern and far enough away to reduce the effect of the assumptions in the last year. For example, for N.S. plaice over the period 1971-1996 one might use the years 1988-1993 (though other ranges could easily be justified).

```
apply(selpat[as.character(1988:1993),],2,mean)
```

3.8.3 Handout

The above procedure can be elaborated in the following R script, which uses the "cohortN()" function described earlier.

<http://tutor-web.net/fish/fish5104vpa/lecture30/annualselectionpattern.r>

When this is used an interesting phenomenon is seen in that the selection pattern appears to have a maximum at an intermediate age. For this particular stock it has sometimes been assumed that the selection pattern on intermediate ages is higher than on the oldest age. This can occur as a consequence of a number of factors, for example the existence of a refuge to which older fish can escape.

3.9 Cohort analysis using F as a starting point

Assume a value for F :

$$N_{ay} = \frac{C_{ay}}{\frac{F_{ay}}{Z_{ay}}(1 - e^{-Z_{ay}})}$$

NB: Now use the same number of ages and years as in the data.

3.9.1 Details

By analogy with VPA, this method sometimes uses fishing mortality rates as a starting point instead of stock sizes at a point in a cohort's history. In this case one uses the cohort analysis based on fishing mortalities equation to obtain the stock size in numbers based on the assumed F .

Definition 3.6. Cohort analysis based on fishing mortalities:

$$N_{ay} = \frac{C_{ay}}{\frac{F_{ay}}{Z_{ay}}(1 - e^{-Z_{ay}})}$$

In this case one normally assumes a value of fishing mortality for the oldest age in the catch data and for the last data year so the stock-numbers matrix becomes the same size as the catch-numbers matrix.

What is now needed is an objective method to choose these initial values of fishing mortality.

3.9.2 Examples

Example 3.9. <http://tutor-web.net/fish/fish5104vpa/lecture30/cohotyanalysisusingf.r>

3.10 The oldest fish

Need assumptions for oldest fish: Use mean across some ages within year

Need to check the fishing pattern first

3.10.1 Details

Recall that fishing mortality rates for the oldest fish and in the last year are not determined by back-calculation.

The most common method for estimating a fishing mortality on the oldest is to assume that fishing mortality rates for the oldest fish are an average of the fishing mortality rates for some younger age groups during the same year.

Notice that this implies that the fishing mortalities have to be constant on the older ages. Therefore the selection pattern is usually investigated in order to decide on the range of ages to be used when computing the mean F to be inserted as the oldest age F .

3.11 The last year

Need assumptions for the last year:

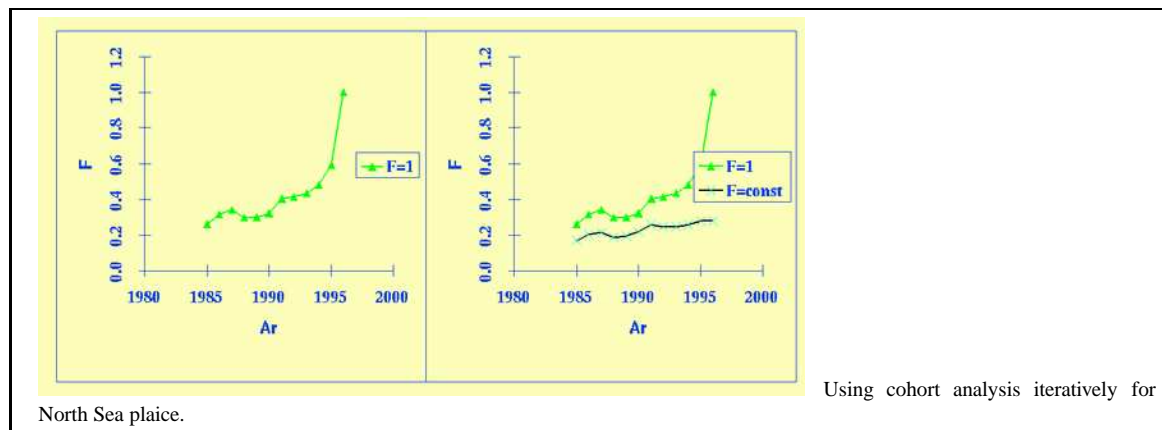
- Use average F over time
- Check (average) fishing pattern
- Repeat VPA with mean F *pattern

3.11.1 Details

Recall that fishing mortality rates for the oldest fish and in the last year are not determined by back-calculation.

For the last year it is commonly assumed that the overall fishing mortality rate in that year is equal to the average of fishing mortality in some previous years. It is usually an improvement to also assume that the selection pattern in the last year is an average of the selection pattern in earlier years.

3.12 Cohort analysis



3.12.1 Details

An overall assessment method based on catches in numbers alone can therefore proceed as follows:

1. Fishing mortality rates for the last year are assumed
2. These mortality rates together with the catch in numbers are used to compute the stock size at the beginning of the last year based on the catch equation.
3. The stock numbers at age for the penultimate year are computed using the cohort analysis equation. This gives the stock size for all age groups except the oldest.
4. The fishing mortality rates for all age groups but the oldest in the penultimate year is computed from the basis of stock sizes of the yearclass in the last two years.
5. The fishing mortality rates for the oldest age are computed as the average of a few younger ages. The inverted catch equation is used to compute the corresponding stock numbers in the oldest age group.
6. The above computations are continued until the tables have been completed
7. An average annual fishing mortality is computed and plotted as a time trend.
8. A table of selection patterns is computed by dividing the annual average fishing mortality into the table of fishing mortalities.
9. An average selection pattern is computed based on recent (but not the last) years.
10. The assumed (relative) fishing mortality by age in the last year is compared with the assumed selection pattern and revised if needed.
11. The overall fishing mortality in the last year is viewed in light of the time trend and revised if needed.

3.12.2 Examples

Example 3.10. North Sea plaice:

First test $F=1$ and then use average f from a few previous years.

3.12.3 Assignment

Assignment 3.1. You start by selecting a stock. Each one of you should ideally have a different stock. You should each hand in a separate and independent analysis.

The project is as follows:

I Initial assumptions

- 1 Assume M
- 2 Assume a fishing mortality for each age in the final year and also for the oldest age in each year. Also compute Z for the last year.

II Some tests

- 1 Compute the stock size in numbers at age in the last year.
- 2 First try by hand to compute one year backwards and then compute the entire table backwards using a loop.
- 3 Compute a corresponding table of fishing mortalities.

III The assessment

- 1 Set up initial assumptions and use $\text{cohortF}()$ to back-calculate the stock.
- 2 Select age groups and compute the average F per year. Plot against time. Conclude how average F in the last year could be improved.
- 3 Compute the selection pattern, which is the F-table divided by the average F . Compute an average pattern over a few years. Plot the average pattern against age. Conclude what the pattern should look like in the last year.
- 4 Revise F in the last year using the average pattern and revised average F . Revise F on the oldest fish using a sensible average across some age groups. Now you can repeat all of 1-6 using improved assumptions.
- 5 Repeat this as needed...

IV Report

Write a report on the development of the stock. Note that such a report needs to describe in considerable detail the methodology used as well as the results, such as the time trend in fishing mortality and biomass along with the estimated selection pattern etc.