# fish5105yieldrec Yield per recruit analysis 

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## 1 Introduction

### 1.1 Background - the development of cohorts in numbers



### 1.1.1 Details

It is well-known that cohorts of fish enter fisheries at some age, usually their numbers in catches first increase and subsequently these fish die.

This is easily seen by looking at catches by age for different cohorts. Initially these catches increase as a function of age as the fish enter the fisheries, i.e. enter the fishing ground or become more accessible to the fishing gear.

### 1.1.2 Examples

Example 1.1. The figure indicates the development in the catches at age of specific cohorts of cod in Icelandic waters. Each curve is a separate cohort. The x-axis indicates at which age the fish were caught.

The figure shows that fish are caught in low numbers at age 3 and most of the catches occur between ages 4 and 6 .

This figure alone does not indicate whether this is a sensible utilization scheme or not.
Importantly, it is seen here that for this cod stock, the catch of the 1983 year-class at age 4 is about twice that of the same year-class at age 6 .

### 1.2 Mean weight at age in catch



### 1.2.1 Details

In order to obtain some sense of whether a stock is being utilized in a rational manner, the mean weight at age in catches needs to be taken into account.

### 1.2.2 Examples

Example 1.2. The mean weight at age for cod in catches is given in the figure. Several interesting observations can be made from this figure. First, growth is considerable from ages 4 to 6 which is the age range when the fish are caught in greatest numbers. Second, the mean weight at age is quite variable from year to year. This will need to be considered when predictions are made.

### 1.3 I-cod: Catch in weight

$\square$

### 1.3.1 Details

When weight at age in catches is multiplied by catches at age the resulting catches in weight at age are quite illustrative.

### 1.3.2 Examples

Example 1.3. The figure indicates the catch biomass by age for different cohorts of cod in Icelandic waters. This picture shows a considerably different story from the graph of numbers caught.

Comparing the two figures of numbers and biomass raises the obvious question, "Were these year-classes utilized sensibly?".

### 1.4 The development of a cohort in numbers



### 1.4.1 Details

The following analyses will all assume that there is no migration in or out of the region of interest. Once a cohort has been born, the numbers can therefore only decrease.

This decline in cohort abundance can be directly observed from surveys, where attempts are made to catch all age groups equally.

### 1.5 Yield per recruit

### 1.5.1 Details

Typical cohort assumptions:

- Year-class size fixed
- I.e. $M$ not variable after this age
- Numbers are reduced due to fishing and natural causes
- The numbers do not increase after the cohort is born
- The individuals gain weight with age
- Assuming the fish have enough to eat
- The biomass of the year-class will reach a maximum
- Weight * numbers =low initially, increases and ends up as zero
- hence there is an age where the year-class would be killed in aquaculture
- Fishing is for many year-classes at once
- So need to estimate what proportion should be caught at each age


### 1.6 Development of yield and biomass



### 1.6.1 Details

When a stock is utilized in such a way that only the very oldest and largest individuals are caught, it is possible that the catch will be below potential because very few individuals can attain such an age and size simply due to natural mortality.

On the other hand, it is not desirable either (from the viewpoint of obtaining high catches) to harvest mainly small fish whose potential growth has not been obtained. This can be demonstrated by comparing fishing with aquaculture. After a year-class is born individual growth continues for most of its life-span. The total biomass increases initially since individual growth exceeds mortality. This trend will, however, not go on for ever. As the individuals become older, their growth tends to slow, at least when compared with natural mortality, and in the end they all die. At one particular point in time, the total weight of the year-class reaches a peak and would then give the greatest yield if the entire year-class can be harvested at once.

The analysis of potential yield from a given cohort is thus of considerable interest. This is done by taking a fixed number of recruits and tracking their development in time, as they are subjected to the usual processes of growth, fishing mortality, and natural mortality.

Several assumptions are needed for this type of analysis:

- Year-class sizes must be fixed (and $M$ is not variable)
- Numbers are reduced due to fishing and natural causes
- The individuals gain weight with age

The first of these assumptions carries with it some implicit considerations. Although it is simple enough to state that one is merely tracking a pre-specified group of fish, this implies that it is sensible to do so. If the survival of such a group of fish would be highly variable due to external causes such as cannibalism or environmental fluctuations, then these computations do not make much sense. It is therefore an implicit assumption that the fish considered are old enough that one can assume that natural mortality, $M$, is not variable,
i.e. fairly stable after this age.

Since migration is assumed not to be of interest the second assumption follows.
The individuals will of course gain weight with age if they have enough to eat. Since a fixed cohort is considered, a fixed set of mean weights at age are considered, typically an average over some years. Effects of this assumption will need to be investigated, both in the form of potential competition effects at high numbers and in the form of variable mean weight at age due to environmental variability in general.

These assumptions will lead to the biomass of the year-class reaching a maximum at some age: Weight * population size will be initially low, increase, and then end up as zero.

Formally, the simple aquaculture analysis is basically

- Look at a single year-class
- Consider harvesting it all at once
- Numbers develop from year to year according to: $N_{a+1}=e^{-M} N_{a}$
- Biomass at a given age is $B_{a}=w_{a} N_{a}$
- Should harvest when biomass is at a maximum (but note cost is missing)

Note that the cost is omitted here (as in most other cases). The effect of including the cost of feed will be fairly obvious (i.e. the harvest is taken earlier).

In any real fishing situation, fishing is for many year-classes at once so there is a need to estimate what proportion should be caught at each age. More specifically, it is not possible, in reality, to simply state that all fish should be caught at a pre-specified age, but gear regulations can affect what proportion is caught at each age and overall effort (or quota) regulations can affect the overall or average mortality inflicted on the cohort.

### 1.6.2 Examples

Example 1.4. The figure indicates the biomass development of a year-class for saithe in Icelandic waters, based on typical values for mean weight at age and natural mortality.

### 1.7 Yield per recruit

### 1.7.1 Details

Although it is simple enough to track the biomass of an unfished cohort, to get indications of the appropriate age to harvest it, fishing gear always catches several age groups together. In order to model this, one must consider the parctical selection of different age groups. The simplest way to do this is by tracking a single cohort in time and analyzing how they grow and die each year.

For a given year-class it is then possible to compute the yield of the year-class.

### 1.7.2 Examples

## Example 1.5.

1. Start with 1000 recruits (youngest age group, e.g. 3)

- Mean weight at age is known
- Know fishing mortality at each age
- Natural mortality is known

2. Compute catch in numbers at age three
3. Compute catch in weight at age three
4. Subtract natural mortality
5. Gives survivors at end of year
6. =number at age 4

### 1.8 Yield per recruit first age simplified example

### 1.8. 1 Details

To predict total yield from a year-class, it is useful to start by making assumptions on how much is caught at the youngest age, what the weight is at that age and how many die at the youngest age.

These numbers form the basis for computing the amount in numbers and weight caught from this age group and in turn the survivors.

A simplified version of the computations could proceed by assuming proportional catches and natural mortality, operating separately.

### 1.8.2 Examples

## Example 1.6.

1. Start with 1000 recruiting cod at age 3

- Mean weight 1.1 kg
- Catch 5
- Natural mortality 18

2. $1000 \times 0.05=50$ cod caught at age 3
3. $50 \times 1.1=55 \mathrm{~kg}$ catch in weight at age 3
4. $1000-50=950$ survive fishing
5. $950 \times 0.82=779$ survive the year
6. =number of 4 year olds at the beginning of next year

### 1.9 Yield per recruit second age group, simplified example

### 1.9.1 Examples

## Example 1.7.

1. $779 \operatorname{cod} 4$ year old

- Mean weight 1.6 kg
- Catch $29 \%$ of age 4 cod
- Natural mortality $18 \%$ p.a.

2. $779 \times 0.29=226$ cod caught at age 4
3. $226 \times 1.6=361 \mathrm{~kg}$ of catch at age 4
4. 779-226=553 fish survive fishing
5. $553 \times 0.82=453$ fish survive the year

Which gives the number of 5 year olds at the beginning of next year.

## 2 Yield per recruit analysis

### 2.1 Introduction

- Initial assessment gives total mortality ( $Z$ )
- Assume a value for natural mortality ( $M$ )
- Assume we have an idea of the fishing mortality by age group
- Can, for example, use back-calculation to get $F$ or selection pattern
- Want to use these to compute expected yield for fixed year-class size


### 2.1.1 Details

The issue of estimating or predicting yield potential from a stock can be separated into two parts: On the one hand there is the expected number of recruits for a given stock size and on the other hand, the yield per each recruit.

An important factor that needs to be considered in relation to the utilization of a fish stock is, therefore, how much can be caught of a specific year-class when its size by number has
been established. In order to examine this, the potential catch per recruit is computed as a function of various assumptions.

The following will assume that there is a specific fishing pattern for a given year-class. The pattern may be changed by modifying mesh size, for example, and the effects of this can easily be verified.

Remember, natural mortality affects the size of the year-class and individual growth affects their mean weight.

### 2.2 Yield per recruit input data

Use $F$ and $s_{a}$ to get $F_{a}=F \cdot s_{a}$.
Note: Assume equilibrium or track a cohort...

### 2.2.1 Details

We will consider a specific year-class and track its development in time.
To determine yield per recruit the following data is needed: fishing mortality at age, natural mortality at age, and weight at age. In addition, the proportion mature at age will be used at a later stage.

In order to simplify computations, it will be assumed that fishing mortality is written in the form $F_{a}=F \cdot s_{a}$.

### 2.2.2 Examples

Example 2.1. Data on cod in Icelandic waters, selection pattern $\left(s_{a}\right)$, natural mortality $\left(M_{a}\right)$ and mean weight at age $\left(w_{a}\right)$. For later use we also note the vector of maturity at age ( $p$ ).
http://tutor-web.net/fish/fish5105yieldrec/lecture20/icod.dat
Now we need to consider the overall $F$ and use $F_{a}=F \cdot s_{a}$.

### 2.3 Simulating a stock

Simulations are useful for testing the effects of assumptions.
Can use vonB with length-weight relationship to get weight at age.
Can use curves for selection and maturity at age.

### 2.3.1 Example

Example 2.2. A simulated stock can be generated using the following commands: http://tutor-web.net/fish/fish5105yieldrec/lecture20/simstock.r

### 2.4 First year yield and stock size

$$
\begin{gathered}
C_{1}=\frac{F_{1}}{Z_{1}}\left(1-e^{-Z_{1}}\right) N_{1} \\
Y_{1}=w_{1} C_{1} \\
N_{2}=N_{1} e^{-Z_{1}} \\
\hline
\end{gathered}
$$

### 2.4.1 Details

Assume that fish enter the fishery at age one. Thus, at the beginning of the first year, the year-class size, $N_{1}$, is simply the number of recruits, i.e. $N_{1}=R$.

The mean weight at age 1 can now be used to compute the biomass of the year-class at age 1 as $B_{1}=w_{1} N_{1}$.

The catch in weight during the first year is determined by fishing mortality, year-class strength and mean weight. Usually one would first compute the catch in numbers.

## Definition 2.1. catch equation:

$$
C_{1}=\frac{F_{1}}{Z_{1}}\left(1-e^{-Z_{1}}\right) N_{1}
$$

Subsequently one would calculate the catch in weight.

Definition 2.2. Catch in weight equation:

$$
Y_{1}=w_{1} C_{1} .
$$

### 2.4.2 Examples

Example 2.3. Suppose the vector $S_{a}$ is the selection pattern, $w$ is the weight at age, and $R$ is the number of recruits. Then the typical first-year computations in R could be:

Fmult<-0. 5
Fmort<-Fmult*sa[1]
Z<-Fmort+M[1]
N1<-R

```
C1<-(Fmort/Z)*(1-exp (-Z))*N1
w1<-w[1]
Y1<-w1*C1
N2<-N1*exp(-Z)
```


### 2.5 Second year

$$
\begin{gathered}
C_{2}=\frac{F_{2}}{Z_{2}}\left(1-e^{-Z_{2}}\right) N_{2} \\
Y_{2}=w_{1} C_{2} \\
N_{3}=N_{2} e^{-Z_{2}}
\end{gathered}
$$

### 2.5.1 Details

During the next year the numbers change according to the stock equation and become $N_{2}=N_{1} e^{-Z_{1}}$, whereas the weight of the year-class becomes $B_{2}=w_{2} N_{2}$ and the catch in numbers becomes

$$
C_{2}=\frac{F_{2}}{Z_{2}}\left(1-e^{-Z_{2}}\right) N_{2}
$$

followed by the catch in weight:

$$
Y_{2}=w_{2} C_{2} .
$$

It is important to note that the number of two-year-old fish stands in direct relation to the number in the previous year. Thus, it is possible to rewrite the equation for two-year-old fish in the catch in terms of the number of one-year-old recruits and mortality coefficients in between.

## Definition 2.3. 2-year-old catch equation:

$$
C_{2}=\frac{F_{2}}{Z_{2}}\left(1-e^{-Z_{2}}\right) e^{-Z_{1}} R .
$$

Thus, there is no real need to re-evaluate the stock in numbers at each step.

### 2.5.2 Examples

Example 2.4. Continuing the earlier example, typical second-year computations in R could be:

```
Fmort<-Fmult*sa[2]
```

Z<-Fmort+M[2]
C2<-(Fmort/Z)*(1-exp (-Z)) *N2
w2<-w [2]
Y2<-w2*C2
N3<-N2*exp (-Z)

### 2.6 The entire sequence

Catch of age, $a$, fish: $C_{a}=\frac{F_{a}}{Z_{a}}\left(1-e^{-Z_{a}}\right) e^{-\sum_{a^{\prime}<a} Z_{a^{\prime}}} R$
Year-class yield: $Y=\sum_{a} Y_{a}=\sum_{a} \frac{F_{a}}{Z_{a}}\left(1-e^{-Z_{a}}\right) w_{a} e^{-\sum_{a^{\prime}<a} Z_{a^{\prime}}} R$.
Yield per recruit: $Y / R=\sum_{a} \frac{F_{a}}{Z_{a}}\left(1-e^{-Z_{a}}\right) w_{a} e^{-\sum_{a^{\prime}<a} Z_{a^{\prime}}}$

### 2.6.1 Details

In general, the catch in numbers of age $a$ fish is given by the catch at age equation.

## Definition 2.4. Catch at age equation:

$$
C_{a}=\frac{F_{a}}{Z_{a}}\left(1-e^{-Z_{a}}\right) e^{-\sum_{a^{\prime}<a} Z_{a^{\prime}}} R
$$

And the catch in weight of age $a$ fish is given by the catch by weight at age equation.

Definition 2.5. Catch by weight at age equation:

$$
Y_{a}=\frac{F_{a}}{Z_{a}}\left(1-e^{-Z_{a}}\right) w_{a} e^{-\sum_{a^{\prime}<a} Z_{a^{\prime}}} R
$$

The total yield of the year-class can also be calculated.

Definition 2.6. Yield per year-class equation:

$$
Y=\sum_{a} Y_{a}=\sum_{a} \frac{F_{a}}{Z_{a}}\left(1-e^{-Z_{a}}\right) w_{a} e^{-\sum_{a^{\prime}<a} Z_{a^{\prime}}} R
$$

An important aspect of the yield per year-class equation is that the total catch from a specific year-class is simply a multiple of the year-class strength and otherwise only depends on the effort directed at each year-class and mean weight by age. From the above, it should also be clear that yield per recruit can be calculated using the yield per recruit equation.

Definition 2.7. Yield per recruit equation

$$
Y / R=\sum_{a} \frac{F_{a}}{Z_{a}}\left(1-e^{-Z_{a}}\right) w_{a} e^{-\sum_{a^{\prime}<a} Z_{a^{\prime}}}
$$

This gives an equation that can be used to assess the effect of a specific fishing pattern and total fishing mortality (multiplier) on the yield from a particular year-class. The resulting quantity is known as the yield per recruit.

The above model is of course a theoretical one. It can, however, give valuable indications as to whether a heavy or light effort is more feasible (or rational) and whether it is better to catch young or old fish.

### 2.6.2 Examples

Example 2.5. It should now be noted, that the stock numbers which are in $N 1, N 2$, $N 3$, can be written in terms of the original recruitment and cumulative $Z$-values. The R function for accumulation is cumsum.
cumsum(3:1)
356
Similarly one can set up a vector of cumulative $Z$-values, first by computing the individual
$Z$-values and then adding them up:
Fmort<-Fmult*sa
Z<-Fmort+M
prop<-(Fmort/Z)*(1-exp(-Z))
Ztemp<-c(0,Z[1: (length (Z)-1)])
cumZ<-cumsum (Ztemp)
C<-prop*exp(-cumZ)
Y<-sum (w*C)
Using an $F$ multiplier (Fmult) of 1 and the values used earlier for cod in Icelandic waters gives $Y / R=1.58$.

### 2.7 Results 1



### 2.7.1 Details

Total catches are of primary interest.

### 2.7.2 Examples

Example 2.6. Consider the following application of the above theory, where we compute the yield per recruit for an $F$ multiplier of 1 .

Fmult<-1
Fmort<-Fmult*sa
Z<-Fmort+M
prop<-(Fmort/Z)*(1-exp(-Z))
Ztemp<-c (0, Z[1: (length (Z)-1)])
cumZ<-exp (-cumsum (Ztemp))
C<-prop*cumZ
$\mathrm{Y}<-\operatorname{sum}(\mathrm{w} * \mathrm{C})$
cbind (Fmort, Z, C=round (C*1000))
This gives the following results:
Fmort Z C
$3 \quad 0.07 \quad 0.27 \quad 61$
$4 \quad 0.33 \quad 0.53196$
$5 \quad 0.60 \quad 0.80 \quad 186$
$6 \quad 0.86 \quad 1.06 \quad 107$
$\begin{array}{llll}7 & 1.09 & 1.29 & 43\end{array}$
$\begin{array}{lllll}8 & 1.15 & 1.35 & 12\end{array}$
$9 \quad 1.15 \quad 1.35 \quad 3$
$\begin{array}{llll}10 & 1.15 & 1.35 & 1\end{array}$
$\begin{array}{llll}11 & 1.15 & 1.35 & 0\end{array}$
$12 \quad 1.151 .35 \quad 0$
$\begin{array}{llll}13 & 1.15 & 1.35 & 0\end{array}$
$14 \quad 1.15 \quad 1.35 \quad 0$
As mentioned earlier, this gives $Y / R=1.58$. It should be noted that most of the catches are from ages 3-7.

### 2.8 Results 2: Reduced effort



### 2.8.1 Details

With this approach is is quite easy to examine the effect of varying effort (through the multiplier) while maintaining the same fishing pattern and investigating separately the effect of modifying the fishing pattern or other assumptions.

### 2.8.2 Examples

Example 2.7. Consider the following application of the above theory, where we compute the yield per recruit for an $F$ multiplier of 0.5 .
Fmult<-0. 5
Fmort<-Fmult*sa
Z<-Fmort+M
prop<-(Fmort/Z)*(1-exp(-Z))
Ztemp<-c (0, Z[1: (length (Z)-1)])
cumZ<-exp (-cumsum (Ztemp))
C<-prop*cumZ
Y<-sum (w*C)
names (Fmort) <-as. character (3:14)
cbind (Fmort, Z, C=round ( $\mathrm{C} * 1000$ ) )
This gives the following results:

|  | Fmort | Z | C |
| :--- | ---: | ---: | ---: |
| 3 | 0.035 | 0.235 | 31 |
| 4 | 0.165 | 0.365 | 109 |
| 5 | 0.300 | 0.500 | 130 |
| 6 | 0.430 | 0.630 | 106 |
| 7 | 0.545 | 0.745 | 68 |
| 8 | 0.575 | 0.775 | 34 |
| 9 | 0.575 | 0.775 | 16 |
| 10 | 0.575 | 0.775 | 7 |
| 11 | 0.575 | 0.775 | 3 |
| 12 | 0.575 | 0.775 | 2 |
| 13 | 0.575 | 0.775 | 1 |
| 14 | 0.575 | 0.775 | 0 |

Here we obtain $Y / R=1.72 \mathrm{~kg}$ for $F=0.5$. Notice that this is slightly higher than earlier results with $F=1.0$. More interestingly, however, is the age composition. In the results for a lower fishing mortality there are many more older fish than for the high fishing mortality.

### 2.9 The yield per recruit curve



### 2.9.1 Details

A thorough understanding of all the underlying assumptions is important when using these methods to evaluate the potential yield from a year-class.

If there are uncertainties associated with the underlying assumptions, then the effects of changing these assumptions should be investigated. Often this does not make much difference, but at other times even the slightest modification can have wide-ranging consequences. In this case the factors in question require closer examination.

This graph shows that each year-class will only produce a finite yield. If the fishing mortality increased too much then the catch will simply include more juvenile fish which have only partially realized their growth potential.

### 2.9.2 Examples

Example 2.8. If we define an R function to return the $Y / R$ for a given value of fishing mortality, then this function could be called with yrfun (0.4) and so on to get the yield per recruit for individual values of $F$.

Alternatively, if one is careful in the function definition, one can obtain a vector of $Y / R-$ values and plot an entire curve of yield-per-recruit for different $F$-values, with
Fvec<- $(0: 150 / 100)$
yr<-sapply (Fvec, yrfun, M, sa, wa)
plot(Fvec,yr,type='l', xlab="F", ylab="Y/R $\mathrm{R}_{\sqcup}(\mathrm{kg})$ ")

### 2.9.3 Handout

The simplistic approach for defining the $Y / R$ function is to use

```
yrfun<-function(Fmult){
    Fmort<-Fmult*sa
    Z<-Fmort+M
    prop<-(Fmort/Z)*(1-exp(-Z))
    Ztemp<-c(0,Z[1:(length(Z)-1)])
    cumZ<-cumsum(Ztemp)
    C<-prop*exp(-cumZ)
    Y<-sum(w*C)
    return(Y)
}
```

but this assumes that the user has already defined the data vectors $M, S_{a}$, and $w_{a}$. In fact no other names can be used.

A much better approach is to define an R function with

```
yrfun<-function(Fmult,M,sa,wa){
    Fmort<-Fmult*sa
    Z<-Fmort+M
    prop<-(Fmort/Z)*(1-exp(-Z))
```

```
    Ztemp<-c(0,Z[1:(length(Z)-1)])
    cumZ<-exp(-cumsum(Ztemp))
    C<-prop*cumZ
    Y<-sum(w*C)
    return(Y)
}
```

so this function can be called with yrfun( $0.7, \mathrm{M}, \mathrm{sa}$, wa) etc.
Notice that the function uses vector arithmetic internally. One can not call this function with a vector of $F$-values and expect to get the correct results.

Note also that the function does not assume the existence of data vectors such as the selection pattern, natural mortality, and weight at age. Rather, they are all passed as arguments.

### 2.10 Yield per recruit and age composition



The effect of variable fishing mortality on yield-per-recruit (solid line) and the age composition of catches at low and high fishing mortality (histograms).

### 2.10.1 Details

A thorough understanding of all the underlying assumptions is important when using these methods to evaluate the potential yield from a year-class.

If there are uncertainties associated with the underlying assumptions, then the effects of changing these assumptions should be investigated. Often this does not make much difference, but at other times even the slightest modification can have wide-ranging consequences. In this case the factors in question require closer examination.

This graph shows that each year-class will only produce a finite yield. If the fishing mortality in increased too much then the catch will simply include more juvenile fish which have only partially realized their growth potential.

### 2.10.2 Examples

Example 2.9. Start with some data, put it into file base95.dat:
Base95.dat: http://tutor-web.net/fish/fish5105yieldrec/lecture20/base95.dat
Use the commands found in the link below to generate a yield-per-recruit at age plot in R:
http://tutor-web.net/fish/fish5105yieldrec/lecture20/yield-per-recruit-age-plot.r

### 2.10.3 Handout

As noted earlier, the simplest Y/R-computations define an R function with:

```
yrfun<-function(Fmult){
    Fmort<-Fmult*sa
    Z<-Fmort+M
    prop<-(Fmort/Z)*(1-exp(-Z))
    Ztemp<-c(0,Z[1:(length(Z)-1)])
    cumZ<-exp(-cumsum(Ztemp))
    C<-prop*cumZ
    Y<-sum(w*C)
    return(Y)
}
```

so this function can be called with yrfun(0.7) etc.
Notice that the function uses vector arithmetic internally. One can therefore not call this function with a vector of $F$-values and expect to get the correct results.

Note also that the function assumes the existence of several data such as the selection pattern, natural mortality and weight at age. It is much "cleaner"to pass such arguments explicitly to the function, as done in the following.

The simulated stock:
http://tutor-web.net/fish/fish5105yieldrec/lecture20/simstock.r
The script yr. is:
http://tutor-web.net/fish/fish5105yieldrec/lecture20/yr.r
Running the Y/R-analysis is done by:
http://tutor-web.net/fish/fish5105yieldrec/lecture20/yield-per-recruit-analysis-for-a-simulatedstock.r

## $2.11 \quad$ F0. 1



### 2.11.1 Details

At zero fishing mortality, $F=0$ corresponds to no fishing and of course this leads to zero catch. When fishing mortality is increased from zero the catches increase dramatically, as implied by a steep slope of the yield-per-recruit curve at the origin. As $F$ increases, the slope gets reduced, until it becomes zero at $F=F_{\max }$.

Note 2.1. The fishing mortality at which the slope of the Y/R-curve has become one tenth of the slope at the origin is denoted $F_{0.1}$.
At a fishing mortality of $F_{0.1}$ the marginal gain in yield is only $10 \%$ of the initial marginal gain when fishing is started.

This should be put in an economic context, where it is clear that taking costs into account will lead to an economically optimum fishing mortality below $F_{\max }$. The $F_{0.1}$ fishing mortality may be thought of as a simple approximation to the economically optimal point on the curve.

### 2.11.2 Examples

Example 2.10. Typical R commands for computing $F_{0.1}$ :
http://tutor-web.net/fish/fish5105yieldrec/lecture20/f01.r

## 3 Spawning stock biomass per recruit

### 3.1 Introduction



### 3.1.1 Details

In the same way as it is possible to compute yield-per-recruit, it is also possible to compute spawning-stock-per-recruit.

Computations of yield-per-recruit and spawning-stock-per-recruit can easily be carried out using a spreadsheet or an interactive computational package such as R. A single recruit can be taken as a starting point for computing the catch in numbers and weight as well as the number of remaining individuals at the end of the year. The same method is then employed to compute the catch for the next year, and so on. Yield-per-recruit and spawning-stock-per-recruit is then computed as the sum of the above-mentioned numbers by age.

### 3.1.2 Examples

Example 3.1. The R functions for calculating spawning-stock-per-recruit: http://tutor-web.net/fish/fish5105yieldrec/lecture30/spawning-stock-per-recruit.r

For the simulated stock seen earlier the following commands can be used to generate a figure:

Fvec<-(0:150/100)
sr<-sapply (Fvec, srfun, M, sa, wa, pa)
plot(Fvec,sr,type='l',xlab="F", ylab="S/Rப(kg)")

### 3.2 A typical representation



### 3.2.1 Details

Yield-per-recruit and spawning-stock-per-recruit are usually depicted together on a graph, as seen above.

### 3.2.2 Examples

Example 3.2. Continuing with the simulated stock, the following R commands combine the two curves in one plot.
http://tutor-web.net/fish/fish5105yieldrec/lecture30/2-curve-plot.r

### 3.3 Some reference points



### 3.3.1 Details

Note 3.1. Some points on the yield per recruit graph are commonly used as reference fishing mortalities, usually called reference points.

Note 3.2. If the curve has a maximum, the corresponding fishing mortality is termed $F_{\max }$.
Since $F_{\text {max }}$ may not exist and may correspond to a very flat portion of the curve, another common reference point is further to the left, where the marginal increase in catches is lower ( $10 \%$ of the increase at the origin). This latter reference point is denoted $F_{0.1}$.

Note 3.3. $F_{0.1}$ is the fishing mortality reference point at which the slope of the Y/R-curve is one-tenth the slope of the origin slope.

It is of importance to note that when increasing fishing mortality above $F_{0.1}$, the catch will only increase marginally (in the long term) whereas the spawning stock biomass (per recruit) will decrease.

It should be noted that those reference points are solely based on growth and mortality. Thus, if $F_{\max }$ is exceeded, a loss in terms of yield is expected (in the long run) for a given year-class size. This does not take into account a possible reduction in recruitment due to overfishing.

### 3.3.2 Examples

Example 3.3. Typical R commands for computing $F_{0.1}$ : http://tutor-web.net/fish/fish5105yieldrec/lecture30/f01.r

## 4 Yield per recruit case studies

### 4.1 I-cod Yield per recruit



### 4.1.1 Examples

Example 4.1. The yield per recruit curve always starts out at zero for zero fishing mortality and increases towards a maximum level. If it reaches this maximum level, it will subsequently decrease.

Asymptotically, as the fishing mortality increases without bounds, the catch will tend to be the entire cohort in its first year.

### 4.2 Herring, I-SuSp, Yield-Recr



### 4.2.1 Examples

Example 4.2. Increased fishing mortality not only affects yield-per-recruit but it is also reflected in the age composition, as seen here for summer spawning herring in Icelandic waters.

### 4.3 I-cod SSB per recruit

Replacement

- $S / R=$ replacement to SSB
- How much does a year-class provide into the SSB?
- $S=\left.R *(S / R)\right|_{F}$
- If a year-class gives less replacement than was needed for its production...



### 4.3.1 Details

The spawning biomass per recruit curve is of great importance when considering the replacement of the stock. For a given fishing mortality, this curve describes the ratio $S / R$. It follows that for a fixed or average year-class size, it is possible to predict the average spawning stock biomass.

Importantly, if there is information about current levels of spawning stock biomass and current average recruitment levels, then the $S / R$-curve provides information on whether the fishery is sustainable. If the average spawning stock from recent average recruitment and current fishing mortality is lower than the estimated stock at present, then the stock is predicted to decline from present levels.

### 4.4 Yield per recruit: Assumptions

- General: Equilibrium
- Mean weight fixed
- Selection pattern fixed
- No equilibrium exists -> gives a direction
- Mean weights are variable
- depend on environment/stock size
- Need to get the number of recruits separately
- Fixed?
- Related to stock size?
- Function of other variables


### 4.5 Yield per recruit: Virtues

- Gives reasonable utilization with minimum work.
- Appears to work as a harvest control rule if use $F=F_{0.1}$.
- Can compare current $F$ with plausible targets.


### 4.5.1 Details

Yield-per-recruit analysis has considerable benefits. It is simple to use and provides target or reference levels of fishing mortality which appear to work quite well in practice if adhered to. Thus, a harvest control rule based on $F=F_{0.1}$ appears to work quite reasonably, but this assumes that management actually observes this as a target.

### 4.6 Yield per recruit: Problems

- Stock-recruitment relationship missing
- Often uninformative (flat curve)
- Assumptions may be too stiff


### 4.6.1 Details

It should be noted that the model described above does not take into account the possibility that individual growth may depend on density. If the density dependent growth rate is known, it must be taken into consideration since it can result in a relatively lower yield at low mortality rates because the size of the stock increases.

## 5 Effect of assumptions on yield per recruit analysis

### 5.1 Introduction

Should investigate effects of all assumptions

- Natural mortality
- Selectivity

- Weight at age

Remember that the "current location" on the graph also depends on assumptions.


### 5.1.1 Details



Mynd 1: A typical set of assumptions.

Assumptions which need to be investigated include the weights at age, natural mortality at age, and mean weight at age. In particular, these values may vary considerably from year to year, rendering interpretation difficult.

When considering the spawning stock biomass per recruit some care needs to be taken to ensure that these assumptions reflect stock values rather than catch values (which have been used here). Additionally, the proportion mature may be particularly difficult to estimate.

