Formula And Variation

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1. Make r the subject of the formula

$P = \frac{200,000}{(\frac{1+r}{12})^{84}}$

$A) r = 12(\frac{p}{200,000})^{84} - 1$

$B) r = 12(200,000 - p)^{\frac{1}{84}}$

$C) r = 12(\frac{p}{200,000})^{\frac{1}{84}}$

$D) r = \frac{12((200,000^{\frac{1}{84}})-1)}{p}$

The first step is to multiply both sides by $(\frac{1+r}{12})^{84}$

$p(\frac{1+r}{12})^{84} = 200,000$

Divide all through by p

$(\frac{1+r}{12})^{84} = \frac{200,000}{p}$

Find the 84^{th} root

$\frac{1+r}{12} = (\frac{200,000}{p})^{\frac{1}{84}}$

This is the same as:

$\frac{1+r}{12} = (\frac{p}{200,000})^{84}$

Multiply all through by 12

$1 + r = 12(\frac{p}{200,000})^{84}$

To make r the subject of the formula, subtract 1 on both sides

$r = 12(\frac{p}{200,000})^{84} - 1$

2. If y varies directly as x, and $y = 8$ when $x = 7$, find the constant k, of proportionality.

$A)8/7$

$B)56$

$C)7/8$

D)None of the above

Since y varies directly with x we will have:

$y = kx$

Where k is a constant of proportionality. We begin by making k the subject of the formula.

$k = \frac{y}{x}$
Solve for k by inserting the values of y and x.

$k = \frac{8}{7}$

3. The value of y varies directly with x, and $y = 15$ when $x = 3$. Find y when $x = 12$

$A)60$

$B)15/4$

$C)5$

$D)1/5$

Since y varies directly with x we will have:

$y = kx$

k is a constant of proportionality. Plugging in the values of y and x gives:

$15 = 3k$

Divide both sides by 3 to obtain the value of k.

$k = 5$

Our equation of proportionality is thus given by:

$y = 5x$

We replace the value of x in the above equation to obtain the value of y.

$y = 5 * 12 = 60$

$y = 60$

4. If y varies directly with the square of x and $y = 6$ when $x = 9$. What is the value of y when $x = 5$?

$A)\frac{50}{27}$

$B)\frac{10}{3}$

$C)\frac{2}{27}$

$D)\frac{10}{27}$

Since y varies directly with the square of x we will have:
$y = k(x^2)$

Replace the values of y and x:

$6 = k(9^2)$

Find the value of k by dividing all through by 9^2.

$k = \frac{6}{9^2} = \frac{2}{27}$

The equation of proportionality will be given by:

$y = \frac{2}{27}x$

The value of y when $x = 5$ will be given by:

$y = \frac{2}{27} * 5^2 = \frac{50}{27}$

$y = \frac{50}{27}$

5. The amount of money raised during a fundraiser is directly proportional to the number of people who attend. Last month the amount of money raised for 500 attendees was ksh 500000. How much money will be raised if 100 more people attend next month?

$A)ksh600,000$

$B)ksh100,000$

$C)ksh400,000$

$D)ksh500,000$

Let A be the amount of money raised, k be a constant of proportionality and n be the number of people. The equation of proportionality is given by:

$A = kn$

Replace the number of people and the amount raised in the above equation.:

$500,000 = 500k$

Find then value of k by dividing all through by 500.

$k = \frac{500,000}{500} = 1,000$

The equation of proportionality is given by:

$A = 1,000n$

Total number of people attending the following month will be:

$500 + 100 = 600$

The amount of money raised the following month would be:

$A = 1,000 * 600 = 600,000$

$A = 600,000$

6. If y varies inversely as x, and $y = 9$ when $x = 8$, find the constant of proportionality.

A)72

B)$\frac{9}{8}$

C)$\frac{8}{9}$

D)None of the above

Since y is inversely proportional to x the equation of proportionality will be given by:

$y = \frac{k}{x}$

We then substitute the values of y and x:

$9 = \frac{k}{8}$

Multiply all through by 8 to obtain the value of k

$k = 9 * 8 = 72$

7. The value of y varies inversely with x and $y = 12$ when $x = 9$. Find x when $y = 15$.

A)7.2

B)108

C)11.25

D)20

y varies inversely x and so we have:

$y = \frac{k}{x}$

Replace the values of y and x:

$12 = \frac{k}{9}$

$k = 12 * 9 = 108$

The equation of proportionality will be given by:

$y = \frac{108}{x}$

Find the value of x by replacing the value of y:

$15 = \frac{108}{x}$

Multiply all through by x and then divide by 15.

$15 * x = 108$

$x = \frac{108}{15} = 7.2$

$x = 7.2$

8. The water temperature of the Indian Ocean varies inversely with the depth of the water. The deeper you dive, the colder it becomes. At a depth of 1800 meters, the water temperature is 50 Celsius. What is the water temperature at the depth of 500 meters?

$A) 180^0 C$

$B) 50^0 C$

$C) 230^0 C$

$D) 130^0 C$

Let t be the water temperature, k be the constant of proportionality and d be the depth of ocean water. The equation of proportionality will be given by:

$t = \frac{k}{d}$

To solve for k replace the values of t and d.

$50 = \frac{k}{1800}$

$k = 50 * 1800 = 90,000$

The equation of proportionality will be:

$t = \frac{90,000}{d}$

Find the water temperature by replacing the values of d.

$t = \frac{90,000}{500} = 180$

$t = 180^0 C$

9. x varies inversely with z and directly with w^3. Write an equation for the given relationship.

$A) x = \frac{kw^3}{z}$

$B) x = \frac{kz}{w^3}$

$C) x = \frac{kw}{z}$

$D) x = kzw^3$

The equation for the above proportion will be given by:

$x = \frac{kw^3}{z}$

10. The density of an object varies directly with its mass and inversely with its volume, its density is $2.5gm/cm^3$ when the mass is $500g$ and volume is $100cm^3$. What is its density when the mass of the object is $1500g$ and its volume is $400cm^3$?

$A)1.875gm/cm^3$

$B)0.5gm/cm^3$

$C)2.5gm/cm^3$

$D)7.5gm/cm^3$

Let d be the density of an object, m be the mass and v be its volume. Then equation of proportionality will be given by:

$d = \frac{km}{v}$

Replace the values of d, m and v.

$2.5 = \frac{k*500}{100} = k * 5$

$k = \frac{2.5}{5} = \frac{1}{2}$

The equation of variation will be given by:

$d = \frac{m}{2v}$

We now replace the values of m and v.

$d = \frac{1500}{2*400} = 1.875$

$d = 1.875gm/cm^3$

11. The number of minutes needed to solve an exercise set of variation problems varies directly as the number of problems and inversely as the number of people working on the solutions. It takes 4 people 36 minutes to solve 21 problems. How long will it take 6 people to solve 42 problems.

A)48 minutes

B)72 minutes

C)36 minutes

D)108 minutes

Let m be the number of minutes taken to solve an exercise, p be the number of people working on the problem, n be the number of problems and k be the constant of proportionality. The equation of variation will be given by:

$m = \frac{kn}{p}$

Plug in the values of p, n and m

$36 = \frac{k*21}{4}$

$k = \frac{36*4}{21} = \frac{48}{7}$

The equation of proportionality will be given by:

$m = \frac{48n}{7p}$

We then replace the values of p and n.

$m = \frac{48*42}{7*6} = 48$

$m = 48$ minutes

12. If 12 women working 8 hours a day can write 15 books in 32 days, how many days will it take 16 women working 12 hours a day to write 24 books?

A)26

B)25

C)10

D)20

Let t be the length of time:

$t = 12 * 8 * 32 = 3,072$

$\frac{3,072}{15} = 204.8$

It takes a one woman 3,072 hours to write 15 books. This means that it takes one woman 204.8 hours to write 1 book. To write 24 books the length of time required would be:

$204.8 * 24 = 4915.2$

To get the number of days, we find the value d:

$4915.2 = d * 16 * 12$

$d = \frac{4915.2}{16*12} = 25.6$

$d = 26Days$

13. If y varies jointly as x and z, and $y = 12$ when $x = 6$ and $z = 3$, find the constant of proportionality.

$A)\frac{2}{3}$

$B)216$

$C)\frac{3}{2}$

D)6

Let k be a constant of proportionality. Then the equation of variation for the above joint variation is given by:

$y = kxz$

We then substitute the values of x, y and z as follows:
$12 = k * 6 * 3$

$k = \frac{12}{6*3} = \frac{2}{3}$

$k = \frac{2}{3}$

14. The variable z varies jointly with x and y. $z = 21$ when $x = 3$ and $y = -6$. Write the equation that relate x, y and z.

$A)z = \frac{-7}{6}xy$

$B)z = \frac{7}{6}xy$

$C)z = \frac{7x}{6y}$

$D)z = \frac{6}{7xy}$

Let k be a constant, then the equation of the above joint variation will be:

$z = kxy$

We then substitute the values of z, x and y as follows.
$21 = k * 3 * (-6)$

$k = \frac{21}{3*(-6)} = -\frac{7}{6}$

The equation for the above proportionality will be:

$z = \frac{-7}{6}xy$

15. If y varies jointly with x and the square root of z. When $y = 4$ and $x = 6$ then $z = 9$. Find y when $x = 9$ and $z = 16$.

A)8

$B)\frac{32}{3}$

C)32

$D)\frac{256}{3}$

The equation of variation for the above joint variation is given by:

$y = kx^{\frac{1}{2}}$

Substitute the values of y, x and z as follows:

$4 = k * 6 * 9^{\frac{1}{2}}$

$4 = k * 6 * 3$

$k = \frac{4}{6*3} = \frac{4}{18} = \frac{2}{9}$

The equation of the above joint variation will be:

$y = \frac{2}{9}xz^{\frac{1}{2}}$

Substitute the values of x and z again.

$y = \frac{2}{9} * 9 * 16^{\frac{1}{2}}$

$y = \frac{2}{9} * 9 * 4 = 8$

$y = 8$

16. Y varies jointly as x and z and inversely as the square of m.

i) Find the equation of variation when $y = 40$, $x = 5$, $z = 8$ and $m = 15$.

$A)y = 225xz/m^2$

$B)y = 15xz/m^2$

$C)y = 225xz/m$

$D)y = 15xz/m$

For the above variation we have:

$y = \frac{kxz}{m^2}$

We substitute the values of y, x, z and m as follows.

$40 = \frac{k*5*8}{15^2}$

$k = \frac{40*15*15}{5*8} = 225$

The equation of variation will be given by:

$y = \frac{225xz}{m^2}$

ii) Solve for y when $x = 12$, $z = 10$ and $m = 15$.

A)120

B)8

C)225

D)1800

From the above equation of variation, substitute the values of x, z and m as follows:

$y = \frac{225xz}{m^2}$

$y = \frac{225*12*10}{15^2} = 120$

$y = 120$

17. The number of phone calls c, between two towns per day is jointly proportional to the population p, of the two towns and inversely proportional to the square of the distance d^2 between the two towns. The population of Kisumu is about 490,000 and the population of Eldoret 320, 000 and the distance between the two towns is $90km$. The average number of calls between the towns is about 100,000. If the average number of daily phone calls between Eldoret and Nakuru which has a population of 350,000 is about 23,200. Find the distance between the two towns to one decimal place.

$A)169.9km$

$B)180km$

$C)28,879.3km$

$D)170km$

Let k be the constant of proportionality. The equation of the above variation is given by.

$c = \frac{kp}{d^2}$

Find the total population for the two towns, Kisumu and Eldoret town.

$p = 490,000 + 320,000 = 810,000$
Plug in the number of calls c,the distance d between Kisumu and Eldoret town and their total population.

$100,000 = \frac{k*810,000}{90^2}$

$k = \frac{100,000*90^2}{810,000} = 1,000$

The equation of proportionality is is given by:

$c = \frac{1,000p}{d^2}$

The next step is to get the total populations between Nakuru and Eldoret town.

$p = 320,000 + 350,000 = 670,000$

We then substitute the number of calls between Eldoret and Nakuru town and their total population.

$23,200 = \frac{1,000*670,000}{d^2}$

$d^2 = \frac{1,000*670,000}{23,200} = 28,879.31$

$d = 169.939km$

Round off the answer to the nearest one decimal place.

$d = 169.9km$

18. Y varies jointly with x^2 and z^3, and varies inversely with r. What is the the percentage change on y when x and r are doubled while z is halved?

A)75 percent

B)25 percent

C)50 percent

D)30 percent

Let k be the constant of proportionality, then the equation of variation would be:

$y = \frac{kx^2z^3}{r}$

After x and r are doubled and z is halved, the new equation becomes:

$y_1 = \frac{k*(2x)^2*(\frac{1}{2}z)^3}{2r}$

$y_1 = \frac{k*4x^2*\frac{1}{8}z^3}{2r} = \frac{kx^2z^3}{4r}$

Get the percentage change by subtracting y_1 from y and multiplying by 100.
$\frac{\frac{kx^2z^3}{r}-\frac{kx^2z^3}{4r}}{\frac{kx^2z^3}{r}} * 100$

$\frac{\frac{3kx^2z^3}{4r}}{\frac{kx^2z^3}{r}} * 100 = \frac{3kx^2z^3}{4kx^2z^3} * 100$

Canceling out the like terms gives:

$\frac{3}{4} * 100 = 75$
The percentage change therefore equals to 75 percent

19. The area of rectangle is jointly related to its length and width. If the length is increased by 45 percent and the width decreased by 15 percent. What will be the percentage change of area?

A)23.25 percent

B)36.75 percent

C)66.75 percent

D)33.25 percent

Let A be the area of the triangle,l be the length, w be the width and k be the constant of proportionality. Then the equation of proportionality will be given by:

$A = klw$

When l is increased by 45 percent while w is decreased by 15 percent,the new equation of proportionality becomes:

$A_1 = k(1.45l)(0.85w) = 1.2325klw$

$A_1 = 1.2325klw$

To find the percentage change, we subtract A from A_1 and multiply by 100.

$\frac{1.2325klw-klw}{klw} * 100 = \frac{0.2325klw}{klw} * 100 = 0.2325 * 100 = 23.25$

Therefore the percentage change equals to 23.25 percent.