Afleiða andhverfu falls; veldisvísisfallið og breiðbogaföllin

math104-3calc Diffrun og afleiður

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Veldisvísisfallið exp(x)

Ef logrinn er skilgreindur fyrst, þá setjum við

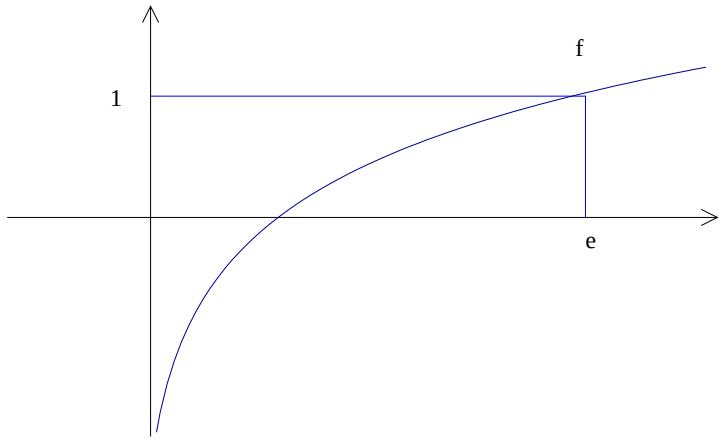
$$exp(x) = \ln^{-1} x$$

og

$$e = \ln^{-1} 1 \quad \Leftrightarrow \quad \ln e = 1$$

og

$$e^x = exp(x)$$



Afleiða andhverfu falls

Ef $y = f(x)$ þar sem f er diffranlegt alls staðar í bili I og $f'(x)$ er **hvergi núll í** I. Þá er f^{-1} diffranlegt alls staðar í $f(I)$, og

$$\left.\frac{d\left(f^{-1}\right)}{dy}\right|_{y=f(a)} = \frac{1}{\left.\frac{df}{dx}\right|_{x=a}}$$

þ.e.

$$(f^{-1})' = \frac{1}{f'}$$

Getum t.d. skrifað $y = f(x), x = f^{-1}(y)$ og þá $f'(x) = \frac{dy}{dx}$ sem gefur

$$(f^{-1})'(y) = \frac{dx}{dy} = 1/\frac{dy}{dx} = \frac{1}{f'(f^{-1}(y))}$$

Skilgreining á e - eyða

Afleiða exp

Afleiða e^x:

$$\frac{d}{dx}e^x = e^x$$

exp sem markgildi

$$\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$$

(sbr. $\lim_{x \to \infty} (1 + \frac{1}{x})^x = e$, P.3).

Almenna veldisvísisfallið

$a > 0, \qquad x \in \mathbb{R}.$

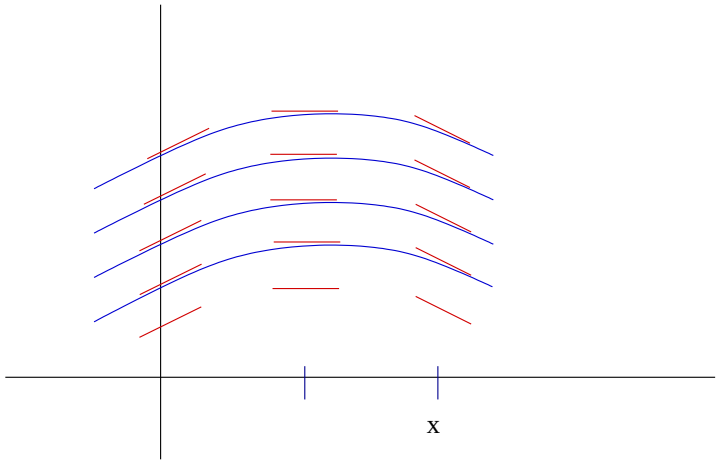
$$a^x = e^{x \ln a} \qquad \left(= e^{\ln a^x} \right)$$

Getum nú skilgreint x^n fyrir öll $x \in \mathbb{R}^+$ og $n \in \mathbb{R}$:

$$x^n = e^{n \ln x}$$

Einfaldar diffurjöfnur

$$\frac{dy}{dx} = 2 \cdot x \cdot y, \quad y(0) = 1$$



Deilum í gegn með y:

$$\frac{1}{y}\frac{dy}{dx} = 2x$$

Heildum m.t.t. x:

$$\int \frac{1}{y(x)}\frac{dy}{dx}\,dx = \int 2x\,dx$$

$$\int \frac{1}{y}\,dy = \int 2x\,dx$$

$$\Rightarrow \qquad \ln y + C_1 = x^2 + C_2$$

Breiðbogaföll - skilgreining

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$
$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$
$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Diffrunarreglur

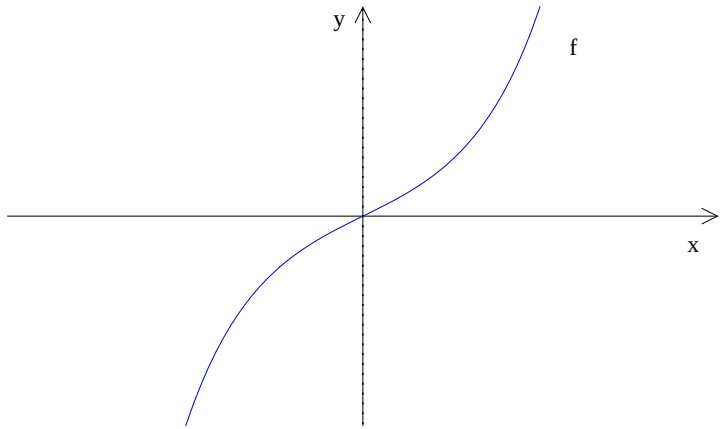
$$\begin{aligned}
\frac{d}{dx}(\cosh x) &= \sinh x \\
\frac{d}{dx}(\sinh x) &= \cosh x \\
\frac{d}{dx}\tanh x &= \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x
\end{aligned}$$

Andhverfur breiðbogafallanna

$$\sinh: \quad \mathbb{R} \to \mathbb{R}$$
$$\sinh^{-1}: \quad \mathbb{R} \to \mathbb{R}$$

$$\cosh: \quad \mathbb{R}^+ \to [1, \infty)$$
$$\cosh^{-1}: \quad [1, \infty) \to \mathbb{R}^+ \quad (= [0, \infty))$$

$$\tanh: \quad \mathbb{R} \to (-1, 1)$$
$$\tanh^{-1}: \quad (-1, 1) \to \mathbb{R}$$



Afleiður breiðbogafalla

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$$