

Heildi og afleiður logra og veldisvísifalla

math104-4calc Heildi

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Skilgreining: Náttúrulegi logrinn

Ein skilgreining á \ln er

$$\ln x \equiv \int_1^x \frac{1}{t} dt \quad x > 0.$$

Ýmsir eiginleikar logra

1

$$\ln 1 = \int_1^1 \frac{1}{t} dt = 0$$

2

$$\ln x = - \int_x^1 \frac{1}{t} dt < 0, \quad 0 < x < 1$$

3

$$\ln x = \int_1^x \frac{1}{t} dt > 0, \quad x > 1$$

4

$$\frac{d}{dx} \ln x = \frac{d}{dx} \int_1^x \frac{1}{t} dt = \frac{1}{x} \quad x > 0$$

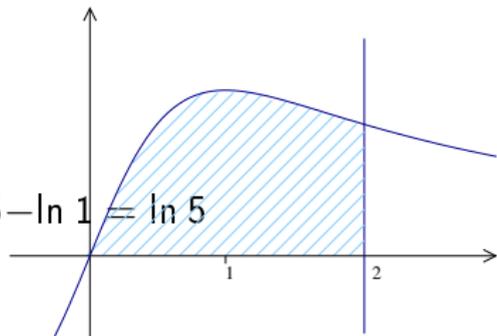
5

$$\frac{d^2}{dx^2} \ln x = -\frac{1}{x^2} < 0$$

Logradæmi

Finnið flatarmálið milli $y = \frac{2x}{x^2+1}$, x -ássi,
 $x = 0$, $x = 2$:

$$A = \int_0^2 \frac{2x}{x^2+1} dx = \underbrace{\int_1^5 \frac{1}{u} du}_{u=x^2+1 \quad du=2x dx} = \ln 5 - \ln 1 = \ln 5$$



Afleiða og stofnfall \exp Afleiða e^x :

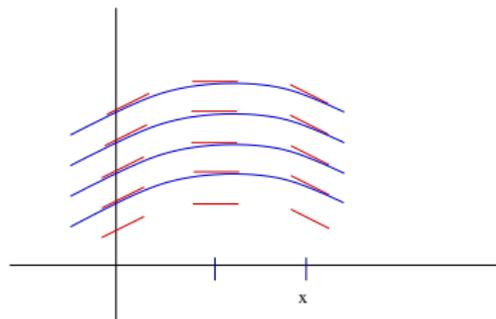
$$\frac{d}{dx} e^x = e^x$$

Stofnfall e^x :

$$\int e^x dx = e^x + C$$

Einfaldar diffurjöfnur

$$\frac{dy}{dx} = 2 \cdot x \cdot y, \quad y(0) = 1$$



Deilum í gegn með y :

$$\frac{1}{y} \frac{dy}{dx} = 2x$$

Heildum m.t.t. x :

$$\int \frac{1}{y(x)} \frac{dy}{dx} dx = \int 2x dx$$

$$\int \frac{1}{y} dy = \int 2x dx$$

$$\Rightarrow \ln y + C_1 = x^2 + C_2$$