Lines as functions of two variables

math121-1-linprog Introduction to linear programming

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The linear objective function

Commonly income, profit or cost are a linear combination of control variables.

$z = ax + by$

Here, z is a function of two variables, x and y. The constants a and b determine the form of the relationship.

The goal will be to find the minimum or maximum of such an objective function, but with some constraints.

The line as a contour

A straight line: $ax + by = c$

A function: $z = ax + by$ (or $z = f(x, y)$ where f is defined by $f(x, y) = ax + by$ for real numbers x, y)

A **contour line** is a set of points where a function takes a constant value: $ax + by = c$ is a contour line for the function $z = ax + by$.

Example: If the unit cost of material A is 3\$ and the unit cost of material B is 4\$ then the cost of purchasing x units of A and y units of B will be $3x + 4y$. The set of all (x, y) values on the line

$$3x + 4y = 25$$

is the set of (x, y)-units, which give a total cost of 25\$.

Moving with the normal vector

If $z = ax + by$ and $z_0 = ax_0 + by_0$, a move from (x_0, y_0), in the direction of the normal vector (a, b) for the line $ax + by = z_0$ will lead to an increase in the value of z:
$(x, y) = (x_0, y_0) + d(a, b)$ with $d > 0$ implies $z > z_0$.