Inverse functions and the logarithm

math612.0 A1: From numbers through algebra to calculus and linear algebra

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Inverse Function

If f is a function, then the function g is the inverse function of f if

$$g(f(x)) = x$$

for all x in which $f(x)$ can be calculated

When the inverse exists: The domain question

Inverses do not always exist. For an inverse of f to exist, f must be one-to-one, i.e. for each x, $f(x)$ must be unique.

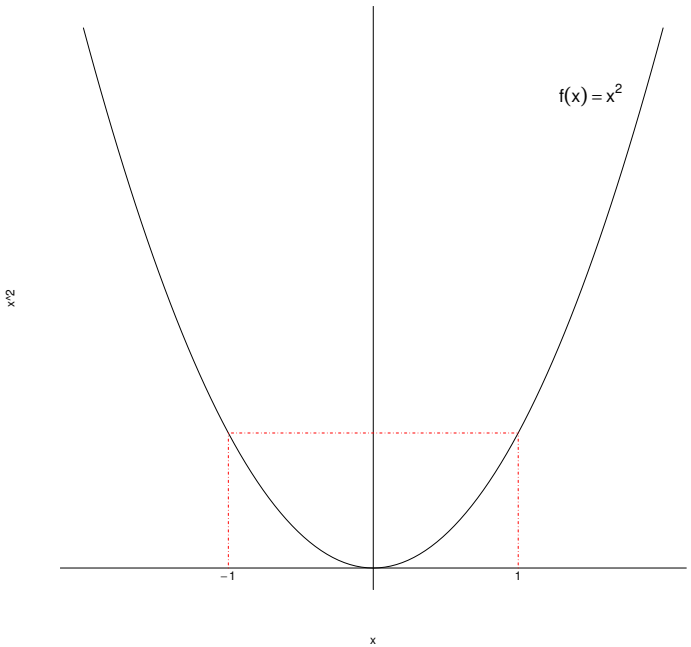

Figure: The function $f(x) = x^2$ does not have an inverse since f(x)=1 has two possible solutions -1 and 1.

The base 10 logarithm

When x is a positive real number in $x = 10^y$, y is referred to as the base 10 logarithm of x and is written as:

$$y = \log_{10}(x)$$

or

$$y = \log(x)$$

The natural logarithm

A logarithm with e as a base is referred to as the natural logarithm and is denoted as ln :

$$y = ln(x)$$

if

$$x = e^y = exp(y)$$

Note that ln is the inverse of exp.

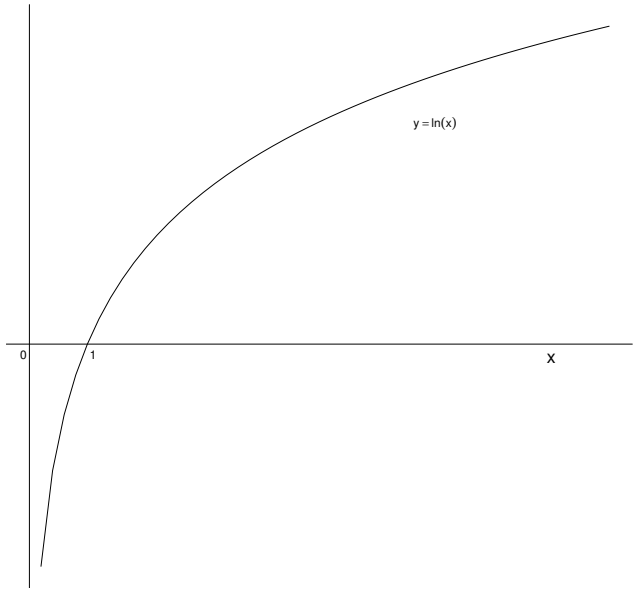

Figure: The curve depicts the fuction $y = \ln(x)$ and shows that ln is the inverse of exp. Note that $\ln(1) = 0$ and when $y = 0$ then $e^0 = 1$.

Properties of logarithm(s)

Logarithms transform multiplicative models into additive models, i.e.

$$\ln(a \cdot b) = \ln a + \ln b$$

The exponential function and the logarithm

The exponential function and the logarithms are inverses of each other

$$x = e^y \Leftrightarrow y = \ln x$$