## Inverse functions and the logarithm math612.0 A1: From numbers through algebra to calculus and linear algebra

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### Inverse Function

#### If f is a function, then the function g is the inverse function of f if

$$g(f(x)) = x$$

for all x in which f(x) can be calculated

### When the inverse exists: The domain question

Inverses do not always exist. For an inverse of f to exist, f must be one-to-one, i.e. for each x, f(x) must be unique.



Figure: The function  $f(x) = x^2$  does not have an inverse since f(x)=1 has two possible solutions -1 and 1.

# The base 10 logarithm

When x is a positive real number in  $x = 10^{y}$ , y is referred to as the base 10 logarithm of x and is written as:

$$y = \log_{10}(x)$$

or

 $y = \log(x)$ 

## The natural logarithm

A logarithm with *e* as a base is referred to as the natural logarithm and is denoted as *In* :

$$y = ln(x)$$

if

$$x = e^y = exp(y)$$

Note that *In* is the inverse of *exp*.

Figure: The curve depicts the fuction  $y = \ln(x)$  and shows that *In* is the inverse of *exp*. Note that  $\ln(1) = 0$  and when y = 0 then  $e^0 = 1$ .



# Properties of logarithm(s)

#### Logarithms transform multiplicative models into additive models, i.e.

$$\ln(a \cdot b) = \ln a + \ln b$$

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# The exponential function and the logarithm

The exponential function and the logarithms are inverses of each other

$$x = e^y \Leftrightarrow y = \ln x$$

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