Continuity and limits

math612.0 A1: From numbers through algebra to calculus and linear algebra

Gunnar Stefansson (editor) with contributions from very many students

March 7, 2022

The concept of continuity

A function is continuous if it has no jumps. Thus, small changes in each x_0, the input, correspond to small changes in the output, $f(x_0)$.

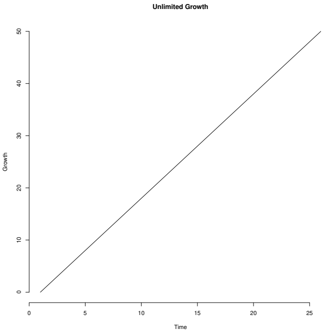
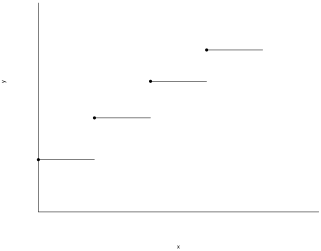


Figure: The above figure is an example of linear growth. Thomas Robert Malthus (1766-1834) warned about the dangers of uninhibited population growth.

Discrete probabilities and cumulative distribution functions

The cumulative distribution function for a discrete random variable is discontinuous.



Notes on discontinuous function

A function is discontinuous for values or ranges of the variable that do not vary continuously as the variable increases. In other words, breaks or jumps.

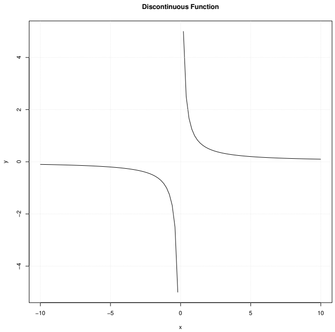
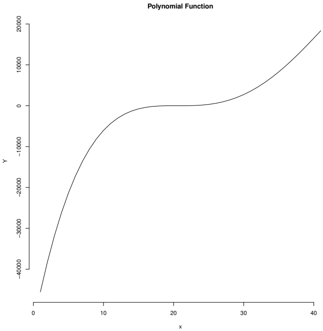


Figure: $f(x) = \frac{1}{x}$, where $x \neq 0$

Continuity of polynomials

All polynomials, $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$, are continuous.



Simple Limits

A "limit" is used to describe the value that a function or sequence "approaches" as the input or index approaches some value. Limits are used to define continuity, derivatives and integrals.

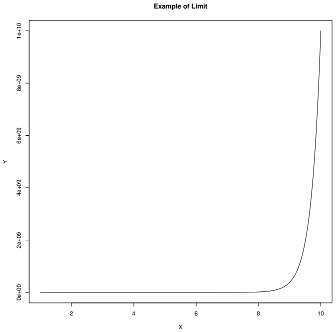


Figure: $f(x) = x^x$, for $x > 0$

More on limits

Limits impose a certain range of values that may be applied to the function.

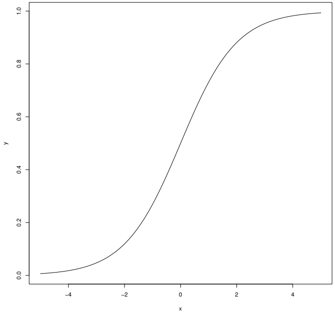


Figure: The function $f(x) = \frac{1}{1+e^{-x}}$.

Example 1:
The Beverton-Holt stock recruitment curve is given by:

$$R = \frac{\alpha S}{1 + \frac{S}{K}}$$

Example 2: A model for proportions:

One-sided limits

$f(x)$ may tend towards different numbers depending on whether $x \to x_0$:
from the right ($x \to x_{0+}$)
or from the left ($x \to x_{0-}$).