Sequences and series

math612.0 A1: From numbers through algebra to calculus and linear algebra

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Sequences

A **sequence** is a string of indexed numbers $a_1, a_2, a_3, \ldots$. We denote this sequence with $(a_n)_{n\geq 1}$.

Convergent sequences

A sequence a_n is said to **converge** to the number b if for every $\varepsilon > 0$ we can find an $N \in \mathbb{N}$ such that $|a_n - b| < \varepsilon$ for all $n \geq N$. We denote this with $\lim_{n\to\infty} a_n = b$ or $a_n \to b$, as $n \to \infty$.

Infinite sums (series)

We are interested in, whether infinite sums of sequences can be defined.

The exponential function and the Poisson distribution

The exponential function can be written as a series (infinite sum):

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

The Poisson distribution is defined by the probabilities

$$p(x) = e^{-\lambda} \frac{\lambda^x}{x!} \text{ for } x = 0,\ 1,\ 2,\ \ldots$$

Relation to expected values

The expected value for the Poisson is given by

$$\begin{aligned}\sum_{x=0}^{\infty} xp(x) &= \sum_{x=0}^{\infty} xe^{-\lambda}\frac{\lambda^x}{x!} \\ &= \lambda\end{aligned}$$