Slopes of lines and curves

math612.0 A1: From numbers through algebra to calculus and linear algebra

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The slope of a line

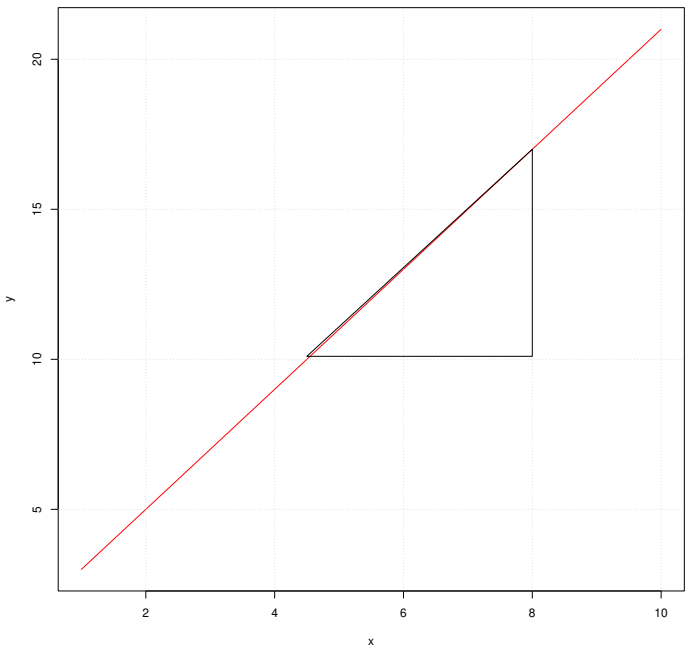
Linear functions produce straight-line graphs. In general, a straight line follows the following equation:

$$y = a + bx,$$

where a and b are fixed numbers.

The line on the graph is the set of points:

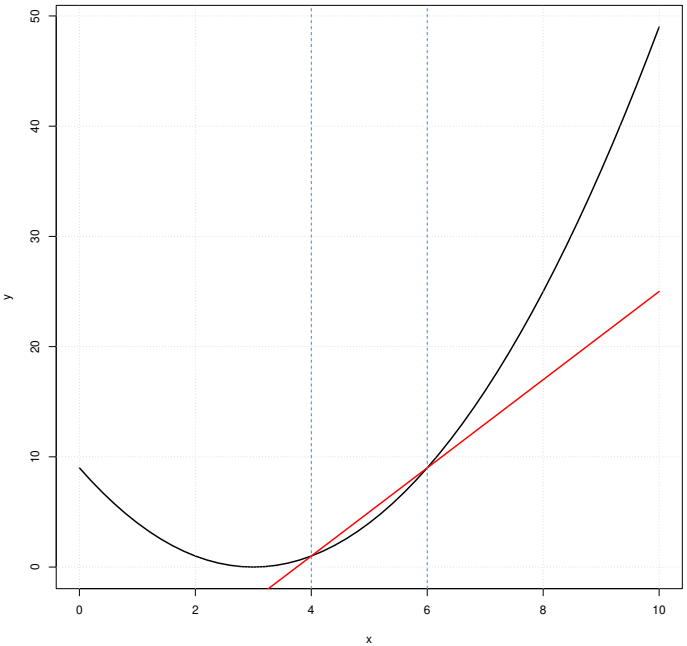
$$\{(x, y) : x, y \in \mathbb{R}, y = a + bx\}.$$



Segment slopes

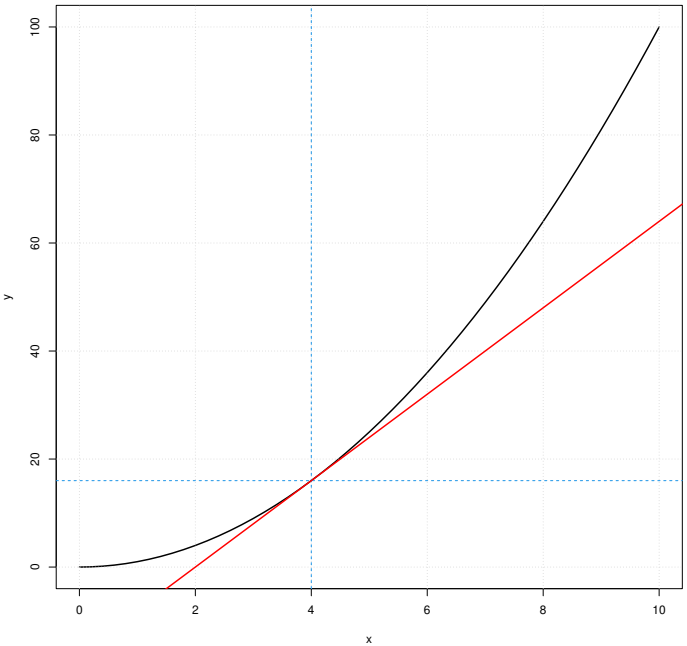
Let's assume we have a more general function
$y = f(x)$.

To find the slope of a line segment, consider 2 x-coordinates, x_0 and x_1, and look at the slope between $(x_0, f(x_0))$ and $(x_1, f(x_1))$.



The slope of $y = x^2$

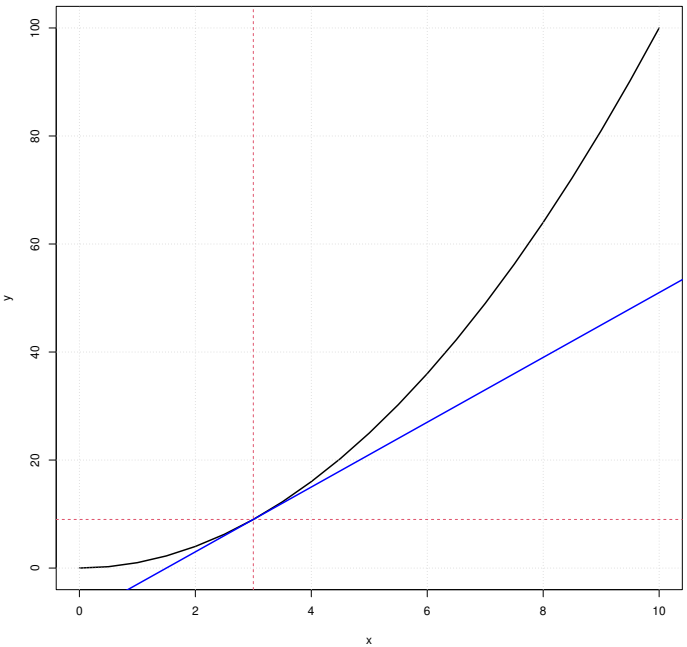
Consider the task of computing the slope of the function $y = x^2$ at a given point.



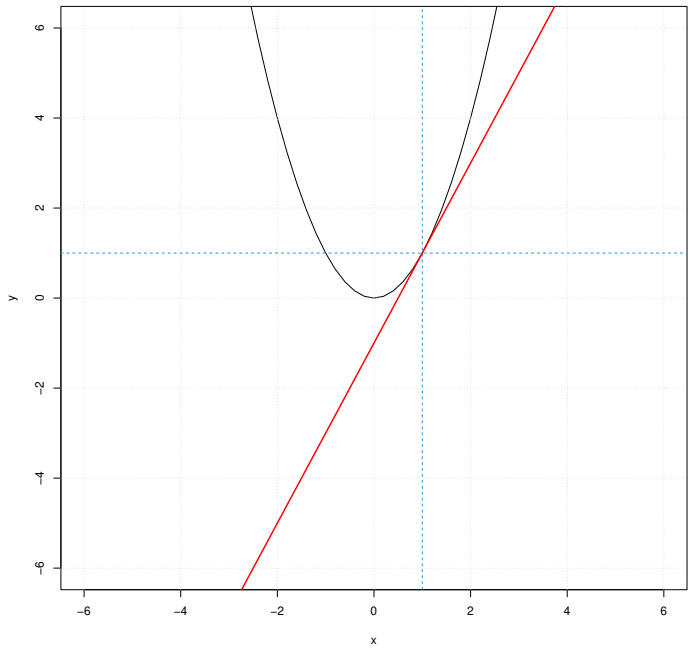
The tangent to a curve

A **tangent** to a curve is a line that intersects the curve at exactly one point. The slope of a tangent for the function $y = f(x)$ at the point $(x_0, f(x_0))$ is

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$



The slope of a general curve



We can have positive slope in the right hand side of the graph and negative slope in the left hand side. The slope is zero at x=0 as shown in the figure 2