Derivatives

math612.0 A1: From numbers through algebra to calculus and linear algebra

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The derivative as a limit

The derivative of the function f at the point x is defined as

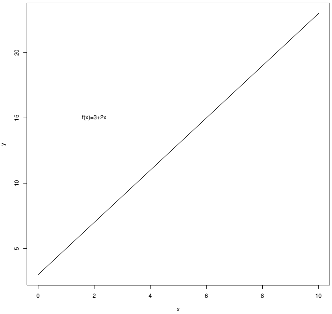
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

if this limit exists.

The derivative of $f(x) = a + bx$

If $f(x) = a + bx$ then $f(x+h) = a + b(x+h) = a + bx + bh$ and thus

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{bh}{h} = b$$



The derivative of $f(x) = x^n$

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$.

The derivative of ln and exp

If

$$f(x) = e^x$$

then

$$f'(x) = e^x$$

If

$$g(x) = \ln(x)$$

then

$$g'(x) = \frac{1}{x}$$

The derivative of a sum and linear combination

If f and g are functions then the derivative of $f+g$ is given by $f'+g'$.

The derivative of a polynomial

The derivative of a polynomial is the sum of the derivatives of the terms of the polynomial.

The derivative of a product

If

$$h(x) = f(x) \cdot g(x)$$

then

$$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Derivatives of composite functions

If f and g are functions and $h = f \circ g$ so that

$h(x) = f(g(x))$ then

$h'(x) = \frac{dh(x)}{dx} = f'(g(x))g'(x)$