Applications of differentiation

math612.0 A1: From numbers through algebra to calculus and linear algebra

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Tracking the sign of the derivative

If f is a function, then the sign of its derivative, f', indicates whether f is increasing ($f' > 0$), decreasing ($f' < 0$), or zero. f' can be zero at points where f has a maximum, minimum, or a saddle point.

Describing extrema using f''

x_0 with $f'(x_0) = 0$ corresponds to a maximum if $f''(x_0) < 0$
x_0 with $f'(x_0) = 0$ corresponds to a minimum if $f''(x_0) > 0$

The likelihood function

If p is the probability mass function (p.m.f.):

$$p(x) = P[X = x]$$

then the joint probability of obtaining a sequence of outcomes from independent sampling is

$$p(x_1) \cdot p(x_2) \cdot p(x_3) \ldots p(x_n)$$

Suppose each probability includes some parameter θ, this is written,

$$p_\theta(x_1), \ldots p_\theta(x_n)$$

If the experiment gives $x_1, x_2 \ldots, x_n$ we can write the probability as a function of the parameters:

$$L_x(\theta) = p_\theta(x_1), \ldots p_\theta(x_n).$$

This is the *likelihood function*.

Plotting the likelihood

missing slide – want to give a numeric example and plot L

Maximum likelihood estimation

If L is a likelihood function for a p.m.f. p_θ, then the value $\hat{\theta}$ which gives the maximum of L:

$$L(\hat{\theta}) = \max_{\theta}(L_\theta)$$

is the maximum likelihood estimator (MLE) of θ

Least squares estimation

Least squares: Estimate the parameters θ by minimizing

$$\sum_{i=1}^{n} (y_i - g_i(\theta))^2$$