Integrals and probability density functions

math612.0 A1: From numbers through algebra to calculus and linear algebra

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Area under a curve

The area under a curve between x=a and x=b (for a positive function) is called the integral of the function.

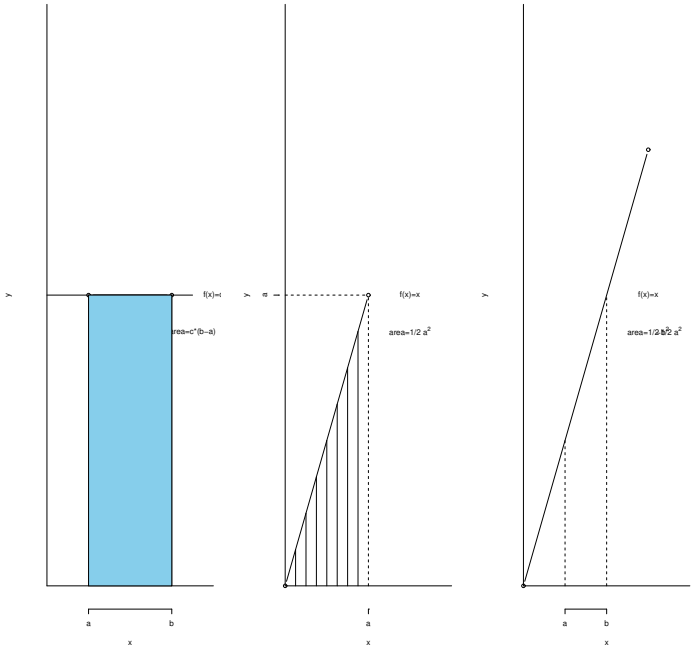


Figure: Example 1, 2 and 3

The antiderivative

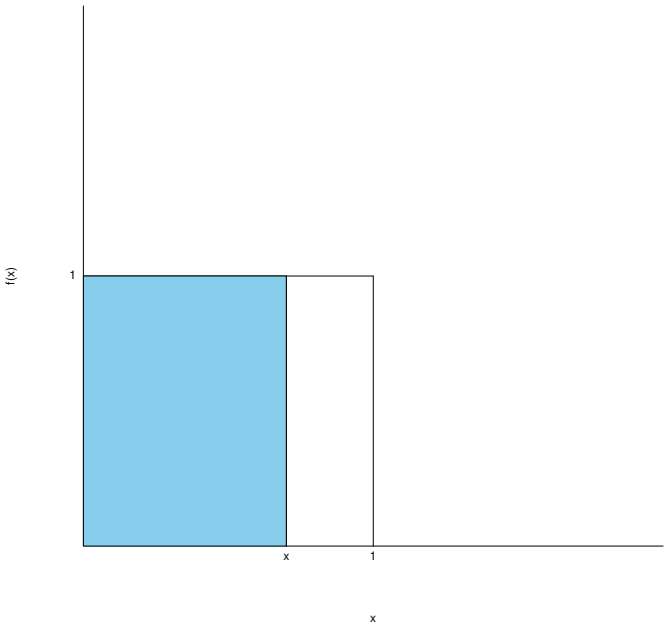
Given a function f, if there is another function F such that $F' = f$, we say that F is the *antiderivative* of f. For a function f the antiderivative is denoted by $\int f dx$.
Note that if F is one antiderivative of f and C is a constant, then $G = F + C$ is also an antiderivative. It is therefore customary to always include the constant, e.g. $\int x dx = \frac{1}{2}x^2 + C$.

The fundamental theorem of calculus

If f is a continuous function, and $F'(x) = f(x)$ for $x \in [a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$

Density functions

The probability density function (p.d.f.) and the cumulative distribution function (c.d.f.).



Probabilities in R: The normal distribution

R has functions to compute values of probability density functions (p.d.f.) and cumulative distribution functions (c.m.d.) for most common distributions.

For example the normal density:

$$p(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$$

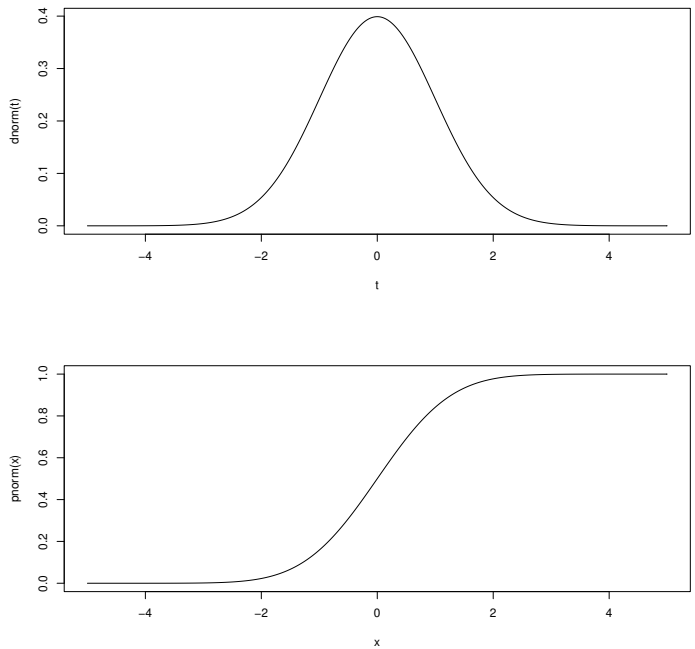


Figure: Top: The probability density function for the normal distribution. Bottom: The cumulative distribution function for the normal distribution.

Some rules of integration

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