Multivariate probability distributions

math612.0 A1: From numbers through algebra to calculus and linear algebra

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Joint probability distribution

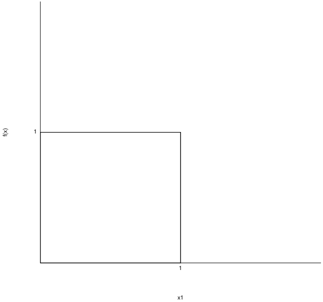
If
$X_1, \ldots, X_n$ are discrete random variables with $P[X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n] = p(x_1, \ldots, x_n)$, where $x_1, \ldots, x_n$ are numbers, then the function p is the joint probability mass function (p.m.f.) for the random variables $X_1, \ldots, X_n$.

For continuous random variables $Y_1, \ldots, Y_n$, a function f is called the joint probability density function if,
$P[Y \in A] = \int \int \ldots \int f(y_1, \ldots y_n) dy_1 dy_2 \cdots dy_n$.

The random sample

A set of random variables $X_1, \ldots, X_n$ is a random sample if they are independent and identically distributed (i.i.d.).

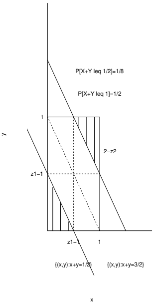
A set of numbers $x_1, \ldots, x_n$ are called a random sample if they can be viewed as an outcome of such random variables.

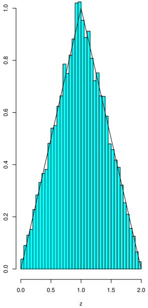


The sum of discrete random variables

The sum of two continuous random variables

If X and Y are continuous random variables with joint p.d.f. f and $Z = X + Y$, then we can find the density of Z by calculating the cumulative distribution function.





Means and variances of linear combinations of independent random variables

If X and Y are random variables and $a, b \in \mathbb{R}$, then

$$E[aX + bY] = aE[X] + bE[Y].$$

Means and variances of linear combinations of measurements

If $x_1, ..., x_n$ and $y_1, ..., y_n$ are numbers, and we set

$$z_i = x_i + y_i$$

$$w_i = ax_i$$

where a>0, then

$$\overline{z} = \frac{1}{n}\sum_{i=1}^{n} z_i = \overline{x} + \overline{y}$$

$$\overline{w} = a\overline{x}$$

$$s_w^2 = \frac{1}{n-1}\sum_{i=1}^{n}(w_i - \overline{w})^2$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}(ax_i - a\overline{x})^2$$

$$= a^2 s^2$$

The joint density of independent normal random variables

If $Z_1, Z_2 \sim n(0,1)$ are independent then they each have density

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, x \in \mathbb{R}$$

and the joint density is the product $f(z_1, z_2) = \phi(z_1)\phi(z_2)$ or

$$f(z_1, z_2) = \frac{1}{(\sqrt{2\pi})^2} e^{\frac{-z_1^2}{2} - \frac{z_2^2}{2}}.$$

More general multivariate probability density functions

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