

Some distributions related to the normal

math612.0 A1: From numbers through algebra to calculus and linear algebra

Gunnar Stefansson (editor) with contributions from very many students

March 7, 2022

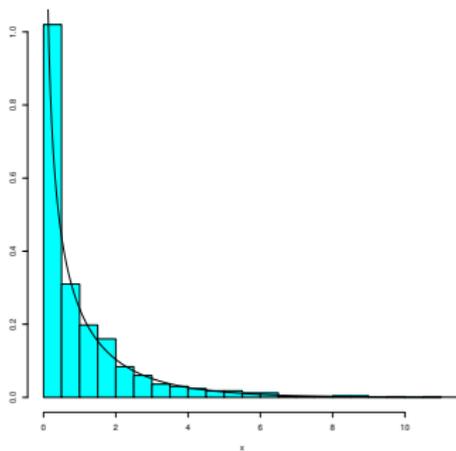
The normal and sums of normals

The sum of independent normally distributed random variables is also normally distributed.

The Chi-square distribution

If $X \sim n(0,1)$, then $Y = X^2$ has a distribution which is called the Chi - square distribution (χ^2) on one degree of freedom. This can be written as:

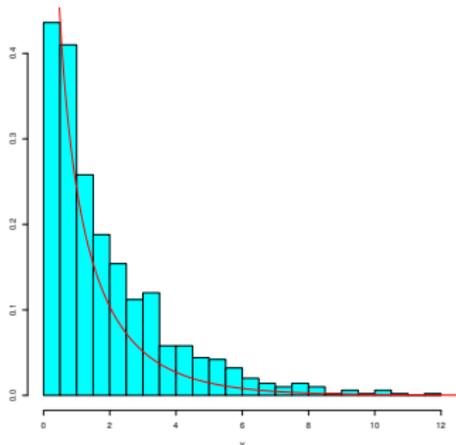
$$Y \sim \chi^2$$



Sum of Chi square Distributions

Let Y_1 and Y_2 be independent variables. If $Y_1 = \chi_{\nu_1}^2$ and $Y_2 = \chi_{\nu_2}^2$, then the sum of these two variables also follows a chi-squared (χ^2) distribution

$$Y_1 + Y_2 = \chi_{\nu_1 + \nu_2}^2$$



Sum of squared deviation

If $X_1, \dots, X_n \sim n(\mu, \sigma^2)$ i.i.d, then

$$\sum_{i=1}^n \left(\frac{X_i - \mu}{\sigma} \right)^2 \sim \chi_n^2,$$

but we are often interested in

$$\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 \sim \chi_{n-1}^2.$$

The t-distribution

If $U \sim n(0, 1)$ and $W \sim \chi_\nu^2$ are independent, then the random variable

$$T = \frac{U}{\sqrt{\frac{W}{\nu}}}$$

has a distribution which we call the t-distribution on ν degrees of freedom denoted $T \sim t_\nu$.

Copyright 2021, Gunnar Stefansson (editor) with contributions from very many students

This work is licensed under the Creative Commons Attribution-ShareAlike License. To view a copy of this license, visit

<http://creativecommons.org/licenses/by-sa/1.0/> or send a letter to Creative Commons, 559 Nathan Abbott Way, Stanford, California 94305, USA.