Some distributions related to the normal

math612.0 A1: From numbers through algebra to calculus and linear algebra

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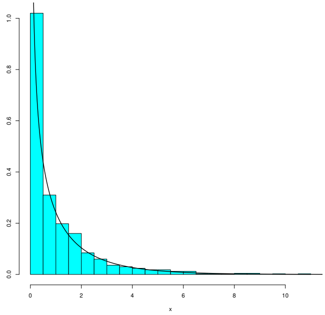
The normal and sums of normals

The sum of independent normally distributed random variables is also normally distributed.

The Chi-square distribution

If $X \sim n\ (0{,}1)$,then $Y = X^2$ has a distribution which is called the Chi - square distribution (χ^2) on one degree of freedom. This can be written as:

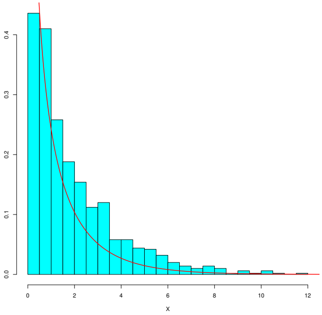
$$Y \sim \chi^2$$



Sum of Chi square Distributions

Let Y_1 and Y_2 be independent variables. If $Y_1 = \chi^2_{\nu_1}$ and $Y_2 = \chi^2_{\nu_2}$,
then the sum of these two variables also follows a chi-squared (χ^2)distribution

$$Y_1 + Y_2 = \chi^2_{\nu_1+\nu_2}$$



Sum of squared deviation

If $X_1, \cdots, X_n \sim n(\mu, \sigma^2)$ i.i.d, then

$$\sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma} \right)^2 \sim \chi^2_n,$$

but we are often interested in

$$\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \sim \chi^2_{n-1}.$$

The t-distribution

If $U \sim n(0,1)$ and $W \sim \chi^2_\nu$ are independent, then the random variable

$$T = \frac{U}{\sqrt{\frac{W}{\nu}}}$$

has a distribution which we call the t-distribution on ν degrees of freedom denoted $T \sim t_\nu$.