Estimation, estimates and estimators

math612.0 A1: From numbers through algebra to calculus and linear algebra

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March 7, 2022

Ordinary least squares for a single mean

If μ is unknown and $x_i, \ldots, x_n$ are data, we can estimate μ by finding

$$\min_{\mu} \sum_{i=1}^{n} (x_i - \mu)^2$$

In this case the resulting estimate is simply

$$\mu = \overline{x}$$

and can easily be derived by setting the derivative to zero.

Maximum likelihood estimation

If $(Y_1, \ldots, Y_n)'$ is a random vector from a density f_θ where θ is an unknown parameter, and y is a vector of observations then we define the **likelihood function** to be

$$L_{\mathrm{y}}(\theta) = f_\theta(y).$$

If, $x_1, \ldots, x_n$ are assumed to come from independent normal distributions with a mean of μ and variance of σ^2, then the joint density is

$$f(x_1) \cdot f(x_2) \cdot \ldots \cdot f(x_n) = \frac{1}{(2\pi)^{n/2}\sigma^n} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$$

and if we assume σ^2 is known then the likelihood function is

$$L(\mu) = \frac{1}{(2\pi)^{n/2}\sigma^n} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$$

Ordinary least squares

Consider the regression problem where we fit a line through (x_i, y_i) pairs with $x_1, \ldots, x_n$ fixed numbers but where y_i is measured with error.

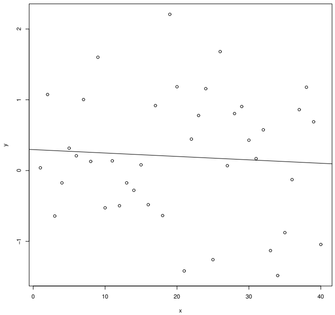

Figure: Regression line through data pairs.

Random variables and outcomes

Estimators and estimates

In OLS regression, note that the values of a and b

$$a = \overline{y} - b\overline{x}$$

$$b = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

are outcomes of random variables e.g. b is the outcome of

$$\hat{\beta} = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

the estimator which has some distribution.

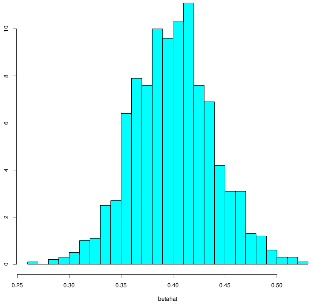

Figure: Shows an example of the distribution of the estimator $\hat{\beta}$