# Some notes on matrices and linear operators math612.0 A1: From numbers through algebra to calculus and linear algebra

Gunnar Stefansson (editor) with contributions from very many students

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## The matrix as a linear operator

#### Let A be an $m \times n$ matrix, the function

$$T_A: \mathbb{R}^n \to \mathbb{R}^m, T_A(\underline{x}) = A\underline{x},$$

is linear, that is

$$T_A(a\underline{x} + b\underline{y}) = aT_A(\underline{x}) + bT_A(\underline{y})$$

if  $\underline{x}, \underline{y} \in \mathbb{R}^n$  and  $a, b \in \mathbb{R}$ .

## Inner products and norms

Assuming x and y are vectors, then we define their inner product by

$$x \cdot y = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$
  
where  $x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$  and  $y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$ 

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# Orthogonal vectors

#### Two vectors x and y are said to be orthogonal if $x \cdot y = 0$ denoted $x \perp y$

## Linear combinations of i.i.d. random variables

Suppose  $X_1, ..., X_n$  are i.i.d. random variables and have mean  $\mu_1, ..., \mu_n$  and variance  $\sigma^2$  then the expected value of Y of the linear combination is

$$Y = \sum a_i X_i$$

and if  $a_1, ..., a_n$  are real constants then the mean is:

$$\mu_{Y} = \sum a_{i}\mu_{i}$$

and the variance is:

$$\sigma^2 = \sum a_i^2 \sigma_i^2$$

# Covariance between linear combinations of i.i.d random variables

Suppose  $Y_1, \ldots, Y_n$  are i.i.d., each with mean  $\mu$  and variance  $\sigma^2$  and  $a, b \in \mathbb{R}^n$ . Writing  $Y = \begin{pmatrix} Y_1 \\ \vdots \\ Y_n \end{pmatrix}$ , consider the linear combination a'Y and b'Y.

## Random vectors

#### $Y = (Y_1, \ldots, Y_n)$ is a random vector if $Y_1, \ldots, Y_n$ are random variables.

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# Transforming random vectors

Suppose

$$\mathsf{Y} = \left(\begin{array}{c} \mathsf{Y}_1\\ \vdots\\ \mathsf{Y}_n \end{array}\right)$$

is a random vector with  $E \mathrm{Y} = \mu$  and  $V \mathrm{Y} = \Sigma$  where the variance-covariance matrix

$$\Sigma = \sigma^2$$

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