Ranks and determinants

math612.0 A1: From numbers through algebra to calculus and linear algebra

Gunnar Stefansson (editor) with contributions from very many students

March 7, 2022

The rank of a matrix

The rank of an nxp matrix, A, is the largest number of columns of A, which are not linearly dependent (i.e. the number of linearly independent columns).

The determinant

Recall that for a 2x2 matrix,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the inverse of A is

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$$

Ranks, inverses and determinants

The following statements are true for an $n \times n$ matrix A:

- $rank(A) = n$
- $det(A) \neq 0$
- A has an inverse