

# The multivariate normal distribution and related topics

math612.0 A1: From numbers through algebra to calculus and linear algebra

Gunnar Stefansson (editor) with contributions from very many students

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# Transformations of random variables

Recall that if  $X$  is a vector of continuous random variables with a joint probability density function and if  $Y = h(X)$  such that  $h$  is a 1-1 function and continuously differentiable with inverse  $g$  so  $X = g(Y)$ , then the density of  $Y$  is given by

$$f_Y(y) = f(g(y))|J|$$

# The multivariate normal distribution

# Univariate normal transforms

The general univariate normal distribution with density

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

is a special case of the multivariate version.

# Transforms to lower dimensions

If  $Y \sim n(\boldsymbol{\mu}, \boldsymbol{\Sigma})$  is a random vector of length  $n$  and  $A$  is an  $m \times n$  matrix of rank  $m \leq n$ , then  $AY \sim n(A\boldsymbol{\mu}, A\boldsymbol{\Sigma}A')$ .

# The OLS estimator

Suppose  $Y \sim n(X\beta, \sigma^2 I)$ . The ordinary least squares estimator, when the  $n \times p$  matrix is of full rank,  $p$ , where  $p \leq n$ , is:

$$\hat{\beta} = (X'X)^{-1}X'Y$$

The random variable which describes the process giving the data and estimate is:

$$b = (X'X)^{-1}X'Y$$

It follows that

$$\hat{\beta} \sim n(\beta, \sigma^2(X'X)^{-1})$$

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