

# The gamma distribution

math612.0 A1: From numbers through algebra to calculus and linear algebra

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# The gamma distribution

If a random variable  $X$  has the density

$$f(x) = \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta^\alpha}$$

where  $x > 0$  for some constants  $\alpha, \beta > 0$ , then  $X$  is said to have a gamma distribution.

# The mean, variance and mgf of the gamma distribution

Suppose  $X \sim G(\alpha, \beta)$  i.e.  $X$  has density

$$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}, x > 0$$

Then,

$$E[X] = \alpha\beta$$

$$M(t) = (1 - \beta t)^{-\alpha}$$

$$V[X] = \alpha\beta^2$$

## Special cases of the gamma distribution: The exponential and chi-squared distributions

Consider the gamma density,

$$f(x) = \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta^\alpha}, x > 0$$

For parameters  $\alpha, \beta > 0$ .

If  $\alpha = 1$  then

$$f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}}, x > 0$$

and this is the density of exponential distribution.

Consider next the case  $\alpha = \frac{\nu}{2}$  and  $\beta = 2$  where  $\nu$  is an integer, so the density becomes,

# The sum of gamma variables

In the general case if  $X_1 \dots X_n \sim G(\alpha, \beta)$  are i.i.d. then  
 $X_1 + X_2 + \dots X_n \sim G(n\alpha, \beta)$ .

In particular, if  $X_1, X_2, \dots, X_v \sim \chi^2$  i.i.d. then  $\sum_{i=1}^v X_i \sim \chi_v^2$ .

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