Probability

(STATS201.stats 201 20: Probability and probability distributions)

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Randomness

- Descriptive statistics describe the sample that we have obtained
- Statistical inference uses the sample to draw conclusions about the whole population.
- The variables that we measure are influenced by some randomness.
- We therefore look at every measurement as a random phenomena.
- In this lecture we look closer at random phenomena.

Events, outcomes and outcome space.

Outcome and outcome space

Every random phenomena has certain possible **outcomes**. The set of all possible outcomes is the **outcome space** an is denoted with Ω.

Event

An **Event** is a particular outcome or a set of particular outcomes of a random phenomena.

Disjoint events

Disjoint events

We say that events A and B are **disjoint** if they contain no common outcome.

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Figure: Disjoint and joint events.

Union and intersection of events

Union of events

The **union** of events A and B is denoted $A \cup B$. It is the set of all outcomes that are in **either** A or B or **both of them**.

Intersection of events

The **Intersection** of events A and B is denoted $A \cap B$. It is the set of al outcomes that are in **both** A and B. If A and B are disjoint, then their intersection is **empty**.

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Figure: Union and intersection.

The complement of an event

Complement of an event

The **Complement** of an event A is denoted A^C. It s the set of all outcomes in Ω that are **not it** A.

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Figure: Complement.

Probability

Probability

The **probability** of a certain outcome of a certain outcome of a random phenomena is the proportion of the cases when that the random phenomena gets that outcome when the phenomena is repeated often enough. This ratio can be at minimum **zero** and at maximum **one**.

Probability of an event

The **probability of an event** A, denoted $P(A)$, is the probability that the observed outcome will be in A.

Equally likely outcomes

Equally likely outcomes

Equally likely outcomes are only defined for random phenomena with **finite** Ω. Then the probability of every outcome in Ω is the same.

Probability of events when all outcomes are equally likely

If all of the outcomes of a random phenomena are **equally likely**, then the probability of an event A are:

$$P(A) = \frac{\text{number of outcomes in } A}{\text{number of outcomes in } \Omega}$$

Formulas

Formulas

1. $P(\Omega) = 1$
2. $P(A^C) = 1 - P(A)$
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
4. If A and B are disjoint, $P(A \cup B) = P(A) + P(B)$

Conditional probability

Conditional probability

With $P(A|B)$ we denote the probability that event A occurs, given that event B has occurred. The probability of $P(A|B)$ can be calculated with

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad \text{if } P(B) > 0.$$

Probability of intersection of events

$$P(A \cap B) = P(A|B)P(B), \quad \text{if } P(B) > 0.$$

Independent events

Independent events

We say that events A and B are **independent** if the probability that an event A occurs does not change even though the event B has occurred and vice versa.

Probability of independent events

If A and B are independent, then

$$P(A \cap B) = P(A) \cdot P(B)$$