Inference on variances

(STATS201.stats 201 30: Statistical inference)

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Introduction

- In this lecture we will discuss inference on the variance of a normally distributed population and how to compare the variances in two normally distributed populations.
- First we will discuss confidence intervals and hypothesis test for the variance of a normally distributed population.
- Then we explore hypothesis tests that can be used when comparing the variances of two normally distributed populations.

Conducting hypothesis tests

Conducting hypothesis tests

1 Decide which hypothesis test is appropriate for our measurements.

2 Decide the α-level.

3 Propose a null hypothesis and decide the direction of the test (one- or two-sided).

4 Calculate the test statistic for the hypothesis test.

5a See whether the test statistic falls within the rejection interval.

5b Look at the p-value of the test statistic.

6 Draw conclusions.

Inference on the variance of a population

- In this section we discuss hypothesis tests and confidence intervals that apply when making inference on the variance of a normally distributed population, σ^2.
- When calculating confidence intervals and testing hypothesis for the variance of a population, the χ^2-distribution is used.
- The null hypothesis in this section is that the variance of the population equals some specific value that we denote σ_0^2.
- The null hypothesis is written $H_0 : \sigma^2 = \sigma_0^2$.
- It depends on the direction of the hypothesis test what conclusion are drawn if the null hypothesis is rejected.
- If the hypothesis test is two-sided we conclude that the variance of the population, σ^2, differs from σ_0^2 but if it is one-sided we can only conclude that the variance is greater or less than σ_0 depending on the case.

Confidence interval for the variance of a population

The lower bound of a $1-\alpha$ confidence interval is: pace-2mm

$$\frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2,(n-1)}}$$

The upper limit of a $1-\alpha$ confidence interval is: pace-2mm

$$\frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2,(n-1)}}$$

The confidence interval can thus be written: pace-2mm

$$\frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2,(n-1)}} \;<\; \sigma^2 \;<\; \frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2,(n-1)}}$$

where n is the number of measurements in the sample and $s^2$is the sample variance. $\chi^2_{1-\alpha/2,(n-1)}$ and $\chi^2_{\alpha/2,(n-1)}$ is found in the χ^2-table.

Inference on the variance of a population

Inference on the variance of a population

The null hypothesis is:

$$H_0 : \sigma^2 = \sigma_0^2$$

The test statistic is:

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

The alternative hypothesis and the rejection areas are:

Alternative hypothesis	Reject H_0 if:
$\mathrm{H}_1 : \sigma^2 < \sigma_0^2$	$\chi^2 < \chi^2_{\alpha,(n-1)}$
$\mathrm{H}_1 : \sigma^2 > \sigma_0^2$	$\chi^2 > \chi^2_{1-\alpha,(n-1)}$
$\mathrm{H}_1 : \sigma^2 \neq \sigma_0^2$	$\chi^2 < \chi^2_{\alpha/2,(n-1)}$ or $\chi^2 > \chi^2_{1-\alpha/2,(n-1)}$

Inference on the variance of two populations

- The hypothesis tests that we discuss in this section are used to compare the variance of two populations that both are normally distributed.
- Tests of this kind are often conducted before hypothesis tests where the means of two populations are compared and the variance of the populations is unknown and the samples are not large.
- The null hypothesis i this section is that the variance of the two populations is equal, written $H_0 : \sigma_1^2 = \sigma_2^2$.
- If the hypothesis test is two-sided we can draw the conclusion that the variances are unequal, but if it is one-sided we can only draw the conclusion that the variance in one sample is greater then the variance in the other sample.

Hypothesis tests for the variances of two populations

Hypothesis tests for the variances of two populations

The null hypothesis is:

$$H_0 : \sigma_1^2 = \sigma_2^2$$

The alternative hypothesis can be one-sided or two-sided and the test-statistic differs by what the alternative hypothesis is. Possible alternative hypothesis, test statistics and their rejection areas are shown below.

Alternative hypothesis	Test statistic	Reject H_0 if:
$\mathrm{H}_1 : \sigma_1^2 < \sigma_2^2$	$F = \frac{S_2^2}{S_1^2}$	$F > F_{1-\alpha,(n_2-1,n_1-1)}$
$\mathrm{H}_1 : \sigma_1^2 > \sigma_2^2$	$F = \frac{S_1^2}{S_2^2}$	$F > F_{1-\alpha,(n_1-1,n_2-1)}$
$\mathrm{H}_1 : \sigma_1^2 \neq \sigma_2^2$	$F = \frac{S_M^2}{S_m^2}$	$F > F_{1-\alpha/2,(n_M-1,n_m-1)}$

In the two-sided test, one shall denote the population with greater sample variance with M and the one with smaller sample variance with m.