Covariance between estimators and inference*

(STATS310.3: Simple linear regression)

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Covariance between estimates of slope and intercept

Need to derive $Cov(\hat{\alpha}\&\hat{\beta})$ for general purposes
Can use this for inference b (for $\hat{Y}$ etc-not line -2.6 waits!) but it is easier to rewrite $\hat{Y}$ as linear combination.

Estimating a point on the regression line

Estimate mean response at x_h:

$$\widehat{E[Y_h]} := \hat{Y}_h = \hat{\alpha} + \hat{\beta} x_h$$

Then

$$E\left[\widehat{E[Y_h]}\right] = E\left[\hat{Y}_h\right] = \alpha + \beta x_h$$

$$Var\left[\widehat{E[Y_h]}\right] = Var[\hat{Y}_h] = \sigma^2\left(\frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum(x_j - \bar{x})^2}\right)$$

Predicting a new observation

Predict Y_h, at x_h
Use $\hat{Y}_h = \hat{\alpha} + \hat{\beta} x_h$
Want d s.t. $P\left[|\hat{Y}_h - Y_h| \leq d\right] = 1 - \alpha$
Old and new are independent:

$$V\left[\hat{Y}_h - Y_h\right] = \sigma^2 \left(1 + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum (x_j - \bar{x})^2}\right)$$

Predicting mean of several new observation

For mean of m new get

$$V\left[\bar{Y}_h - Y_h\right] = \sigma^2\left(\frac{1}{m} + \frac{1}{n} + \frac{(x_h - \bar{x})^2}{\sum(x_j - \bar{x})^2}\right)$$