Distributions of linear projections of vectors of random variables

stats545.1 Theory of linear models

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Linear combinations of independent random variables

$\mathbf{c}$ a column vector
$\mathbf{Y}$ a vector of independent random variables
Same σ, expected values may differ, $E[\mathbf{Y}] = \boldsymbol{\mu}$
Then

$$E\left[\mathbf{c}'\mathbf{Y}\right] = \mathbf{c}'\boldsymbol{\mu}$$
$$V\left[\mathbf{c}'\mathbf{Y}\right] = \mathbf{c}'\mathbf{c}\sigma^2$$

Covariance between linear combinations of independent random variables

$\mathbf{a}$, $\mathbf{b}$ column vectors
$\mathbf{Y}$ a vector of independent random variables
Same σ, expected values may differ, $E[\mathbf{Y}] = \boldsymbol{\mu}$
Then

$$Cov\left[\mathbf{a}'\mathbf{Y}, \mathbf{b}'\mathbf{Y}\right] = \mathbf{a}'\mathbf{b}\sigma^2$$

Linear projections of independent random variables

A an $n \times n$ matrix
Y a vector of n independent random variables, mean $\boldsymbol{\mu}$, $V[Y_i] = \sigma^2$.
Then

$$E\left[\mathbf{AY}\right] = \boldsymbol{\mu}$$

$$V\left[\mathbf{AY}\right] = \mathbf{AA}'\sigma^2$$

$Vc'Y$ and VAY => repeated $Cov(\hat{\alpha})$ and $Cov(\hat{\beta})$

Linear transformations of dependent random variables

$\mathbf{A}$ a matrix
$\mathbf{Y}$ a vector of random variables whose variances and covariances exist as a matrix, $\mathbf{\Sigma} = (\sigma_{ij})$ with $\sigma_{ij} = Cov(Y_i, Y_j)$.
Then

$$V[\mathbf{AY}] = \mathbf{A\Sigma A}'$$

$Vc'Y$ and VAY => repeated $Cov(\hat{\alpha})$ and $Cov(\hat{\beta})$