General properties of linear projections of vectors of random variables

stats545.1 545.1 Point estimation and variances in the linear model

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Linear combinations of independent random variables

c a column vector
Y a vector of independent random variables
Same σ, expected values may differ, $E[\mathrm{Y}] = \boldsymbol{\mu}$
Then

$$E\left[\mathrm{c}'\mathrm{Y}\right] = \mathrm{c}'\boldsymbol{\mu}$$

$$V\left[\mathrm{c}'\mathrm{Y}\right] = \mathrm{c}'\mathrm{c}\sigma^2$$

Covariance between linear combinations of independent random variables

a, b column vectors
Y a vector of independent random variables
Same σ, expected values may differ, $E[\mathrm{Y}] = \boldsymbol{\mu}$
Then

$$Cov\left[\mathrm{a}'\mathrm{Y}, \mathrm{b}'\mathrm{Y}\right] = \mathrm{a}'\mathrm{b}\sigma^2$$

Linear projections of independent random variables

A an $n \times n$ matrix
Y a vector of n independent random variables, mean $\boldsymbol{\mu}$, $V[Y_i] = \sigma^2$.
Then

$$E\left[\mathrm{AY}\right] = \boldsymbol{\mu}$$

$$V\left[\mathrm{AY}\right] = \mathrm{AA}'\sigma^2$$

$Vc'Y$ and VAY => repeated $Cov(\hat{\alpha})$ and $Cov(\hat{\beta})$

Linear combinations of dependent random variables

$\mathrm{a} \in \mathbb{R}^n$ a vector
Y a vector of n random variables whose variances and covariances exist as a matrix, $\Sigma = (\sigma_{ij})$ with $\sigma_{ij} = Cov(Y_i, Y_j)$.
Then

$$V\left[\mathrm{a}'\mathrm{Y}\right] = \mathrm{a}'\Sigma\mathrm{a}$$

Linear transformations of dependent random variables

A a matrix
Y a vector of random variables whose variances and covariances exist as a matrix, $\Sigma = (\sigma_{ij})$ with $\sigma_{ij} = Cov(Y_i, Y_j)$.
Then

$$V\left[\mathrm{AY}\right] = \mathrm{A}\Sigma\mathrm{A}'$$

$Vc'Y$ and VAY => repeated $Cov(\hat{\alpha})$ and $Cov(\hat{\beta})$