

General properties of linear projections of vectors of random variables

stats545.1 545.1 Point estimation and variances in the linear model

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Linear combinations of independent random variables

c a column vector

Y a vector of independent random variables

Same σ , expected values may differ, $E[Y] = \boldsymbol{\mu}$

Then

$$E [c'Y] = c'\boldsymbol{\mu}$$

$$V [c'Y] = c'c\sigma^2$$

Covariance between linear combinations of independent random variables

a, b column vectors

Y a vector of independent random variables

Same σ , expected values may differ, $E[Y] = \mu$

Then

$$\text{Cov} [a'Y, b'Y] = a'b\sigma^2$$

Linear projections of independent random variables

A an $n \times n$ matrix

Y a vector of n independent random variables, mean μ , $V[Y_i] = \sigma^2$.

Then

$$E[AY] = \mu$$

$$V[AY] = AA'\sigma^2$$

$Vc'Y$ and $VAY \Rightarrow$ repeated $Cov(\hat{\alpha})$ and $Cov(\hat{\beta})$

Linear combinations of dependent random variables

$a \in \mathbb{R}^n$ a vector

Y a vector of n random variables whose variances and covariances exist as a matrix, $\Sigma = (\sigma_{ij})$ with $\sigma_{ij} = \text{Cov}(Y_i, Y_j)$.

Then

$$V[a'Y] = a'\Sigma a$$

Linear transformations of dependent random variables

A a matrix

Y a vector of random variables whose variances and covariances exist as a matrix, $\Sigma = (\sigma_{ij})$ with $\sigma_{ij} = \text{Cov}(Y_i, Y_j)$.

Then

$$V[AY] = A\Sigma A'$$

$Vc'Y$ and $VAY \Rightarrow$ repeated $\text{Cov}(\hat{\alpha})$ and $\text{Cov}(\hat{\beta})$

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