Expected values and variances in multiple linear regression

stats545.1 Theory of linear models

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Expected values in multiple linear regression

Expected values in multiple linear regression

$$E[\hat{\boldsymbol{\beta}}] = \boldsymbol{\beta}$$

- only depends on mean structure

Example: Sometimes a dependent variable does not vary in a simple linear fashion as a function of two independent variables as in $EY_i = \alpha + \beta x_i + \gamma w_i$. In particular, it may become obvious that the response, as a function of x, does not have the same slope for two different values of z. In this case an **interaction model** is required: $y_i = \alpha + \beta x_i + \gamma w_i + \delta x_i w_i$. Defining $x_{i1} = 1,\ x_{i2} = x_i,\ x_{i3} = w_i,\ x_{i4} = x_i w_i$, this becomes a multiple linear regression model.

Variances in multiple linear regression

$$
\begin{aligned}
& V\left[\hat{\beta}\right] \\
= \;& V\left[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}\right] \\
= \;& \left((\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\right) V\left[\mathbf{y}\right] \left((\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\right)' \\
= \;& \ldots \\
= \;& \sigma^2(\mathbf{X}'\mathbf{X})^{-1}.
\end{aligned}
$$

Depends on true variance structure - not on p.d.f.

Covariances between parameter estimates

Var-cov matrices also have correlations between estimates.

Also get numerical estimates of the var-cov matrix as well as all correlations once an estimate, $\hat{\sigma}^2$, of σ^2 becomes available.
Example: SLR

References Neter, J., Kutner, M. H., Nachtsheim, C. J. and Wasserman, W. 1996. Applied linear statistical models. McGraw-Hill, Boston. 1408pp.