

Expected values and variances in multiple linear regression

stats545.1 545.1 Point estimation and variances in the linear model

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September 3, 2022

Expected values in multiple linear regression

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If

$$E[y] = X\beta$$

and

$$\hat{\beta} = (X'X)^{-1} X'y$$

then

$$E[\hat{\beta}] = \beta$$

- only depends on mean structure

Example: Sometimes a dependent variable does not vary in a simple linear fashion as a function of two independent variables as in $EY_i = \alpha + \beta x_i + \gamma w_i$. In particular, it may become obvious that the response, as a function of x , does not have the same slope for two different values of z . In this case an **interaction model** is required: $y_i = \alpha + \beta x_i + \gamma w_i + \delta x_i w_i$. Defining $x_{i1} = 1$, $x_{i2} = x_i$, $x_{i3} = w_i$, $x_{i4} = x_i w_i$, this becomes a multiple linear regression model.

Variances in multiple linear regression

If

$$V[y] = \sigma^2 I$$

and

$$\hat{\beta} = (X'X)^{-1} X'y$$

then

$$\begin{aligned} & V[\hat{\beta}] \\ = & V[(X'X)^{-1} X'y] \\ = & ((X'X)^{-1} X') V[y] ((X'X)^{-1} X')' \\ = & \dots \\ = & \sigma^2 (X'X)^{-1}. \end{aligned}$$

Depends on true variance structure - not on p.d.f.

Covariances between parameter estimates

Var-cov matrices also have correlations between estimates.

Also get numerical estimates of the var-cov matrix as well as all correlations once an estimate, $\hat{\sigma}^2$, of σ^2 becomes available.

Example: SLR

References Neter, J., Kutner, M. H., Nachtsheim, C. J. and Wasserman, W. 1996. Applied linear statistical models. McGraw-Hill, Boston. 1408pp.

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