Special cases of Scheffes confidence sets: Applications to simple linear regression

stats545.4 545.4 Multivariate confidence intervals

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The setup

$$y_i \sim n(\alpha + \beta x_i, \sigma^2), i = 1, \ldots, n$$

The intercept

C.I. for α alone is the same as before.

The slope

C.I. for slope is the same as before

A simultaneous confidence set for the slope and intercept

Confidence ellipse in the α - β plane

$$\left\{\psi: \frac{\left(\hat{\psi} - \psi\right)'\mathsf{B}^{-1}\left(\hat{\psi} - \psi\right)/q}{||\mathsf{y} - \mathsf{X}\hat{\boldsymbol{\beta}}||^2/(n-r)} \le F_{q,n-r,1-\alpha}\right\}$$

where

$$\psi = \left(\begin{array}{c} \alpha \\ \beta \end{array}\right)$$

Confidence band for the regression line

Simultaneous band for the entire regression line:

$$\left\{a+bx\pm s\sqrt{2F_{2,n-2,1-\alpha}\left\{\frac{1}{n}+\frac{(x-\bar{x})^2}{\sum_i(x_i-\bar{x})^2}\right\}}:x\in\mathbb{R}\right\}$$

Scheffes confidence intervals giving reg line

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