Orthogonal projections in multiple regression

stats545.2 545.2 The multivariate normal distribution and projections in the linear model

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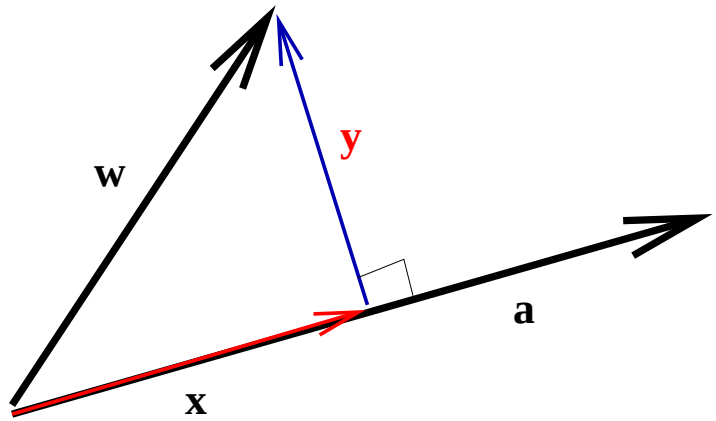
Background to projections

If a is a vector then we can write a general vector w in the form $w = x + y$ where $x = ka$ and $a'y = a \cdot y = 0$.
In the general case,

$$k = \frac{w \cdot a}{||a||^2},$$

and for unit vectors a we obtain

$$k = w \cdot a.$$



Projections and bases

The Gram-Schmidt technique uses projections to iteratively build an orthonormal basis, $u_1, \ldots u_r$ which spans the same space as a sequence of arbitrary starting vectors, $a_1, a_2, \ldots a_p$.
In linear regression, the starting vectors are typically the columns of the X-matrix. r above is then the rank of the matrix.