# Orthogonal projections in multiple regression 

stats545.2 545.2 The multivariate normal distribution and projections in the linear model

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## Background to projections

If $a$ is a vector then we can write a general vector $w$ in the form $w=x+y$ where $x=k a$ and $a^{\prime} y=a \cdot y=0$.
In the general case,

$$
k=\frac{\mathrm{w} \cdot \mathrm{a}}{\|\mathrm{a}\|^{2}}
$$

and for unit vectors a we obtain


$$
k=\mathrm{w} \cdot \mathrm{a}
$$

## Projections and bases

The Gram-Schmidt technique uses projections to iteratively build an orthonormal basis, $u_{1}, \ldots u_{r}$ which spans the same space as a sequence of arbitrary starting vectors, $a_{1}, a_{2}, \ldots a_{p}$.
In linear regression, the starting vectors are typically the columns of the $X$-matrix. $r$ above is then the rank of the matrix.

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