

Estimable functions

stats545.3 545.3 Hypothesis tests in the linear model, model building
and predictions

Gunnar Stefansson

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Estimable functions: The problem

If \mathbf{X} is not of full rank, then the LS problem does not have a unique solution for $\hat{\boldsymbol{\beta}}$.

In general not all combinations of the form $\mathbf{c}'\hat{\boldsymbol{\beta}}$ may have unique solutions.

A linear combination $\mathbf{c}'\boldsymbol{\beta}$ is an **estimable function** if there is a vector of numbers, \mathbf{a} , such that

$$E[\mathbf{a}'\mathbf{y}] = \mathbf{c}'\boldsymbol{\beta}$$

for all $\boldsymbol{\beta}$.

NB: Viewed as a function of the unknown parameter vector, $\boldsymbol{\beta}$.

NB: The E -operator depends on $\boldsymbol{\beta}$, could define $g(\boldsymbol{\beta}) := \mathbf{c}'\boldsymbol{\beta}$ and $h(\boldsymbol{\beta}) := E_{\boldsymbol{\beta}}[\mathbf{a}'\mathbf{y}]$ and require $g(\boldsymbol{\beta}) = h(\boldsymbol{\beta}) \quad \forall \boldsymbol{\beta}$ for some \mathbf{a} .

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}.$$

Classification of estimable functions

Theorem: A parametric function $\psi = \mathbf{c}'\boldsymbol{\beta}$ is estimable if and only if $\mathbf{c}' = \mathbf{a}'\mathbf{X}$ for some $\mathbf{a} \in \mathbf{R}^n$.

Gauss-Markov theorem

Theorem: (Gauss-Markov theorem): Let $EY = X\beta$, $VY = \sigma^2 I$. Then every estimable function $c'\beta$ has a unique unbiased linear estimate which has minimum variance in the class of all unbiased linear estimates. This estimate can be written the form $c'\hat{\beta}$ where $\hat{\beta}$ is any LS estimator.

Testing hypotheses in the linear model

Theorem: If $\underbrace{\mathbf{y}}_{n \times 1} \sim n(\underbrace{\mathbf{X}}_{n \times p} \underbrace{\boldsymbol{\beta}}_{p \times 1}, \sigma^2 \underbrace{\mathbf{I}}_{n \times n})$ and $\hat{\boldsymbol{\psi}}$ is a vector of estimable functions, then $\hat{\boldsymbol{\psi}} \sim n(\boldsymbol{\psi}, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\psi}}})$, $\frac{\|\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}\|^2}{\sigma^2} \sim \chi_{n-r}^2$ and these two quantities are independent.

References Neter, J., Kutner, M. H., Nachtsheim, C. J. and Wasserman, W. 1996. Applied linear statistical models. McGraw-Hill, Boston. 1408pp. Scheffe, H. 1959. The analysis of variance. John Wiley and Sons, Inc, New York. 477pp. **Copyright** 2020, Gunnar Stefansson

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