

# Tests of hypotheses including multiple comparisons in the linear model

stats545.4 545.4 Multivariate confidence intervals

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# On distributions

$$\text{If } \underbrace{\mathbf{y}}_{n \times 1} \sim n(\underbrace{\mathbf{X}}_{n \times p} \underbrace{\boldsymbol{\beta}}_{p \times 1}, \sigma^2 \underbrace{\mathbf{I}}_{n \times n})$$

and  $\boldsymbol{\psi}$  are estimable functions, then

then  $\hat{\boldsymbol{\psi}} \sim n(\boldsymbol{\psi}, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\psi}}})$ ,  $\frac{\|\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}\|^2}{\sigma^2} \sim \chi_{n-r}^2$  and these two quantities are independent.

## Confidence ellipsoids

$$P_{\beta} \left[ (\hat{\psi} - \psi)' \mathbf{B}^{-1} (\hat{\psi} - \psi) \leq qs^2 F_{q,n-r,1-\alpha} \right] = 1 - \alpha$$

This is an example of **simultaneous inference**: a single statement on a multivariate estimable function using a single  $\alpha$ -level.

## Confidence interval for a single estimable function

For a single estimable function with estimator  $\hat{\psi} = \mathbf{c}'\hat{\boldsymbol{\beta}} = \mathbf{a}'\mathbf{y}$ ,

$$\hat{\sigma}_{\hat{\psi}}^2 = \mathbf{a}'\mathbf{a}s^2$$

and

A confidence interval for  $\psi$ : can be based on

$$\left(\hat{\psi} - \psi\right)^2 \leq \mathbf{a}'\mathbf{a}s^2 F_{1,n-r,1-\alpha}$$

or on

$$P \left[ \psi \in \left[ \hat{\psi} - t_{n-r,1-\alpha/2} \sqrt{\mathbf{a}'\mathbf{a}s}, \hat{\psi} + t_{n-r,1-\alpha/2} \sqrt{\mathbf{a}'\mathbf{a}s} \right] \right] = 1 - \alpha$$

# Testing hypotheses for multiple estimable functions

$$H_0 : \psi_1 = \psi_2 = \dots = \psi_q = 0 \text{ vs } H_a : \text{not } H_0$$

Reject  $H_0$  if

$$\hat{\boldsymbol{\psi}}' \mathbf{B}^{-1} \hat{\boldsymbol{\psi}} > qs^2 F_{q,n-r,1-\alpha}$$

# Multiple comparisons

$$P \left[ \hat{\psi}_i - \sqrt{qF_{q,n-r,1-\alpha}} \hat{\sigma}_{\hat{\psi}_i} < \psi_i < \hat{\psi}_i + \sqrt{qF_{q,n-r,1-\alpha}} \hat{\sigma}_{\hat{\psi}_i} \quad i = 1, \dots, q \right] \geq 1 - \alpha$$

# Data-snooping

When  $q = 1$  the S-method is the same as a t-test. When  $q > 1$ , conducting multiple t-tests will ruin the error rate. The S-method permit multiple test:

Can use the S-method for data-snooping

May want to use a large  $\alpha$

Better than LSD: Know explicitly the error rate

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