Tests of hypotheses including multiple comparisons in the linear model

stats545.4 545.4 Multivariate confidence intervals

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On distributions

If $\underbrace{\mathbf{y}}_{n\times 1} \sim n(\underbrace{\mathbf{X}}_{n\times p}\underbrace{\boldsymbol{\beta}}_{p\times 1}, \sigma^2 \underbrace{\mathbf{I}}_{n\times n})$
and $\boldsymbol{\psi}$ are estimable functions, then
then $\hat{\boldsymbol{\psi}} \sim n\left(\boldsymbol{\psi}, \Sigma_{\hat{\boldsymbol{\psi}}}\right)$, $\frac{||\mathbf{y}-\mathbf{X}\hat{\boldsymbol{\beta}}||^2}{\sigma^2} \sim \chi^2_{n-r}$ and these two quantities are independent.

Confidence ellipsoids

$$P_{\boldsymbol{\beta}}\left[\left(\hat{\boldsymbol{\psi}}-\boldsymbol{\psi}\right)'\mathrm{B}^{-1}\left(\hat{\boldsymbol{\psi}}-\boldsymbol{\psi}\right)\leq qs^2F_{q,n-r,1-\alpha}\right]=1-\alpha$$

This is an example of **simultaneous inference**: a single statement on a multivariate estimable function using a single α-level.

Confidence interval for a single estimable function

For a single estimable function with estimator $\hat{\psi} = \mathrm{c}'\hat{\boldsymbol{\beta}} = \mathrm{a}'\mathrm{y}$,

$$\hat{\sigma}^2_{\hat{\psi}} = \mathrm{a}'\mathrm{a}s^2$$

and
A confidence interval for ψ: can be based on

$$\left(\hat{\psi} - \psi\right)^2 \leq \mathrm{a}'\mathrm{a}s^2 F_{1,n-r,1-\alpha}$$

or on

$$P\left[\psi \in \left[\hat{\psi} - t_{n-r,1-\alpha/2}\sqrt{\mathrm{a}'\mathrm{a}}s, \hat{\psi} + t_{n-r,1-\alpha/2}\sqrt{\mathrm{a}'\mathrm{a}}s\right]\right] = 1 - \alpha$$

Testing hypotheses for multiple estimable functions

$$H_0 : \psi_1 = \psi_2 = \ldots = \psi_q = 0 \text{ vs } H_a : \text{ not } H_0$$

Reject H_0 if

$$\hat{\boldsymbol{\psi}}' \mathsf{B}^{-1} \hat{\boldsymbol{\psi}} > qs^2 F_{q,n-r,1-\alpha}$$

Multiple comparisons

$$P\left[\hat{\psi}_i - \sqrt{qF_{q,n-r,1-\alpha}}\hat{\sigma}_{\hat{\psi}_i} < \psi_i < \hat{\psi}_i + \sqrt{qF_{q,n-r,1-\alpha}}\hat{\sigma}_{\hat{\psi}_i} \quad i = 1, \ldots, q\right] \geq 1$$

Data-snooping

When $q = 1$ the S-method is the same as a t-test. When $q > 1$, conducting multiple t-tests will ruin the error rate. The S-method permit multiple test:

Can use the S-method for data-snooping

May want to use a large α

Better than LSD: Know explicitly the error rate