

Tests of hypotheses including multiple comparisons in the linear model

stats545.4 545.4 Multivariate confidence intervals

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On distributions

$$\text{If } \underbrace{y}_{n \times 1} \sim n\left(\underbrace{X}_{n \times p}, \underbrace{\beta}_{p \times 1}, \sigma^2 \underbrace{I}_{n \times n}\right)$$

and ψ are estimable functions, then

then $\hat{\psi} \sim n\left(\psi, \Sigma_{\hat{\psi}}\right)$, $\frac{\|y - X\hat{\beta}\|^2}{\sigma^2} \sim \chi_{n-r}^2$ and these two quantities are independent.

Confidence ellipsoids

$$P_{\beta} \left[(\hat{\psi} - \psi)' B^{-1} (\hat{\psi} - \psi) \leq qs^2 F_{q,n-r,1-\alpha} \right] = 1 - \alpha$$

This is an example of **simultaneous inference**: a single statement on a multivariate estimable function using a single α -level.

Confidence interval for a single estimable function

For a single estimable function with estimator $\hat{\psi} = c'\hat{\beta} = a'y$,

$$\hat{\sigma}_{\hat{\psi}}^2 = a'as^2$$

and

A confidence interval for ψ : can be based on

$$\left(\hat{\psi} - \psi\right)^2 \leq a'as^2 F_{1,n-r,1-\alpha}$$

or on

$$P \left[\psi \in \left[\hat{\psi} - t_{n-r,1-\alpha/2} \sqrt{a'as}, \hat{\psi} + t_{n-r,1-\alpha/2} \sqrt{a'as} \right] \right] = 1 - \alpha$$

Testing hypotheses for multiple estimable functions

$$H_0 : \psi_1 = \psi_2 = \dots = \psi_q = 0 \text{ vs } H_a : \text{not } H_0$$

Reject H_0 if

$$\hat{\psi}' B^{-1} \hat{\psi} > qs^2 F_{q, n-r, 1-\alpha}$$

Multiple comparisons

$$P \left[\hat{\psi}_i - \sqrt{qF_{q,n-r,1-\alpha}} \hat{\sigma}_{\hat{\psi}_i} < \psi_i < \hat{\psi}_i + \sqrt{qF_{q,n-r,1-\alpha}} \hat{\sigma}_{\hat{\psi}_i} \quad i = 1, \dots, q \right] \geq 1 - \alpha$$

Data-snooping

When $q = 1$ the S-method is the same as a t-test. When $q > 1$, conducting multiple t-tests will ruin the error rate. The S-method permit multiple test:

Can use the S-method for data-snooping

May want to use a large α

Better than LSD: Know explicitly the error rate

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