Special cases of Scheffes confidence sets: Applications to simple linear regression

stats545.4 545.4 Multivariate confidence intervals

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The setup

$$y_i \sim n(\alpha + \beta x_i, \sigma^2), \ \ i = 1, \ldots, n$$

The intercept

C.I. for α alone is the same as before.

The slope

C.I. for slope is the same as before

A simultaneous confidence set for the slope and intercept

Confidence ellipse in the α-β plane

$$\left\{ \boldsymbol{\psi} : \frac{\left(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}\right)' \mathsf{B}^{-1} \left(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}\right) / q}{||\mathsf{y} - \mathsf{X}\hat{\boldsymbol{\beta}}||^2/(n-r)} \leq F_{q,n-r,1-\alpha} \right\}$$

where

$$\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Confidence band for the regression line

Simultaneous band for the entire regression line:

$$\left\{ a + bx \pm s\sqrt{2F_{2,n-2,1-\alpha}\left\{\frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_i (x_i - \bar{x})^2}\right\}} : x \in \mathbb{R} \right\}$$

Scheffes confidence intervals giving reg line