

# Introduction to nonlinear statistical models

stats545.5 545.5 Extending the linear model

Gunnar Stefansson

September 3, 2022

# The assumptions - and what goes wrong

$$y \sim n(X\beta, \sigma^2 I)$$

may be wrong.

Simple variance exceptions are easy to handle:

- $Vy_i = u_i\sigma^2$  where  $u_i$  are known
- $\Sigma_y = \sigma^2 B$  where  $B$  is known
- $\Sigma_y$  may contain “a few” unknown parameters

# Maximum likelihood

The MLE is usually a good estimator

Applies to very many estimation problems

Need to specify the complete likelihood function

Can take into account dependence, different variances, non-normality, non-linear response etc

Examples: Gamma mean, fish growth, length-weight relationships.

# Nonlinear least squares

Common model:

$$E y_i = g(x_i, \beta), \quad 1 \leq i \leq n$$

Common estimation method:

$$\min_{\beta} \sum_i (y_i - g(x_i, \beta))^2$$

Also need to estimate uncertainty. Use Hessian or bootstrap.

**Copyright** 2021, Gunnar Stefansson

This work is licensed under the Creative Commons Attribution-ShareAlike License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-sa/1.0/> or send a letter to Creative Commons, 559 Nathan Abbott Way, Stanford, California 94305, USA.