Distributions and expectations in the one-way layout

(STATS545.4: Analyses of variance and covariance)

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Distributions

It is of interest to consider the distributions of various quantities, not only under $H_0 : \mu_1 = \ldots = \mu_I$ but also when H_0 does not hold. Assume, therefore that

$$y_{ij} \sim n(\mu_i, \sigma^2),\ 1 \leq j \leq J_i,\ 1 \leq i \leq I, \text{ i.i.d.}$$

In particular, y_{ij} independent with $Ey_{ij} = \mu_i$ and $Vy_{ij} = \sigma^2$.

We then have $\bar{y}_{i.} = \frac{\sum_j y_{ij}}{J_i}$ with expected value

$$E\left[\bar{y}_{i.}\right] = \mu_i$$

and variance

$$V\left[\bar{y}_{i.}\right] = \sigma^2 / J_i$$

and under normality the estimators $\bar{y}_{i.}$ have the obvious properties

$$\bar{y}_{i.} \sim n(\mu_i, \sigma^2 / J_i)$$

The expected MSR

Can obtain

$$E\left[MSR\right] = \sigma^2 + \frac{\sum_i J_i \left(\mu_i - \mu\right)^2}{I - 1}$$

in one-way layout.