Random and mixed effects models

(STATS546.2: Applied analysis of variance (work in progress))

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Introduction

In the usual ANOVA models we have fixed effects

- Interest is only in those fixed factor levels

In the random effects models we have random effects

- Interest is not only in those factor levels in the study but in the population of all possible factor levels

In the mixed effects models we have both random and fixed effects

- Interest is in those fixed factor levels in the study and sometime in the population of all possible factor levels of the random effect.

Random or fixed effects

When to use random and when to use fixed effects?

We put a factor as a random effects when we are not really interested in the specific levels of that factor and can assume the levels were drawn at random from the population of all possible levels.

We put a factor as a fixed effect when we are only interested in the levels of that factor and not in any other possible levels.

$$
\begin{aligned}
&= \mu + a_i + \epsilon_{ij} \\
a_i &\sim N(0, \sigma_a^2) \\
\epsilon_{ij} &\sim N(0, \sigma^2)
\end{aligned}
$$

Variance components

In a random model with one factor there are two variance components.

The variance σ_a^2 of the random effect a_i and the variance σ^2 of the errors ϵ_{ij}

The variance σ_y^2 of the observations are the sum of the two variance components $\sigma_y^2 = \sigma_a^2 + \sigma^2$.

$$
\begin{aligned}
&= \sigma_a^2 \\
\text{cov}(y_{ij}, y_{lj}) &= 0
\end{aligned}
$$

Correlation between observation belonging to the same group is called intraclass correlation

$$\rho = cor(y_{ij}, y_{ik}) = \frac{\sigma_a^2}{\sigma_a^2 + \sigma^2}$$

$$: \sigma_a^2 = 0$$

$$\mathrm{H}_1 \quad : \sigma_a^2 > 0$$

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

$$H_0 : \rho = \rho_0$$
$$H_1 : \rho \neq \rho_0$$

ANOVA table

The ANOVA table for the random effect model is the same as for the regular ANOVA when the experiment is balanced.

Table: ANOVA table

Source	df	SS	MS
Random effect	$g-1$	$SST = \sum n_i(\bar{y}_i. - \bar{y}_{..})^2$	$MST = SST/(g-1)$
Error	$N-g$	$SSE = \sum_i \sum_j (y_{ij} - \bar{y}_i.)^2$	$MSE = SSE/(N-g)$
Total	$N-1$	$SSTOT = \sum_i \sum_j (y_{ij} - \bar{y}..)^2$	

Estimation of variance components

$$\hat{\sigma}_a^2 = \frac{MST - MSE}{n}$$

Approximate $1 - \alpha$ confidence interval for σ_a^2 is as follows:

$$\frac{df \times \hat{\sigma_a^2}}{\chi^2_{1-\alpha/2,df}} \leq \sigma_a^2 \leq \frac{df \times \hat{\sigma_a^2}}{\chi^2_{\alpha/2,df}}$$

where

$$df = \frac{(n\hat{\sigma_a^2})^2}{\frac{MST^2}{g-1} + \frac{MSE^2}{r(n-1)}}$$

$$\hat{\sigma}^2 = MSE$$

$1 - \alpha$ confidence interval for σ^2 is calculated as follows:

$$\frac{g(n-1)MSE}{\chi^2_{1-\alpha/2,g(n-1)}} \leq \sigma^2 \leq \frac{g(n-1)MSE}{\chi^2_{\alpha/2,g(n-1)}}$$

Estimation of intraclass correlation

$$\hat{\rho} = \frac{\hat{\sigma}_a^2}{\hat{\sigma}_a^2 + \hat{\sigma}^2}$$

$1 - \alpha$ confidence limit for ρ is

$$\frac{L}{1+L} \leq \rho \leq \frac{U}{1+U}$$

where

$$L = \frac{1}{n}\left(\frac{MST}{MSE}\frac{1}{F_{1-\alpha/2,g-1,g(n-1)}} - 1\right)$$

$$U = \frac{1}{n}\left(\frac{MST}{MSE}\frac{1}{F_{\alpha/2,g-1,g(n-1)}} - 1\right)$$

Unequal sample sizes

The methods explained earlier only apply for balanced studies.

When sample sizes are unequal the calculation become more complex.

We use statistical software to do the calculation for us.

Maximum likelihood (ML) or restricted maximum likelihood (REML) are used.

Example - Ram breeding experiment

Breeding experiment at the Agricultural university of Iceland

Carcass weight measured of male lambs

Want to know if variance in carcass weight can be related to the sire.

Unequal sample sizes.

Example - Ram breeding experiment

Use the lme and interval function in R to do the calculations.

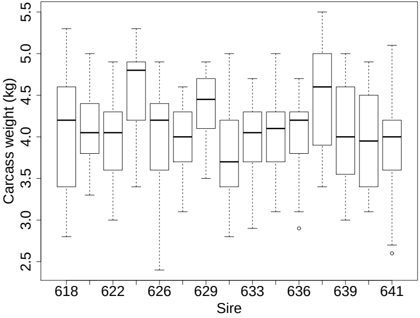
REML used to estimate the variance components.

Confidence intervals obtained using normal approximation to the distribution of the estimates.

We get $\hat{\sigma}_a^2 = 0.010$ and $\sigma^2 = 0.341$ and the 95% confidence limits are:

$$0.0005 \leq \sigma_a^2 \leq 0.177$$

$$0.282 \leq \sigma^2 \leq 0.411$$



Random effects model with two factors

Variance components in the two way layout

Intraclass correlation in the two way layout

Hypothesis

ANOVA table

Estimation of variance components

Estimation of intraclass correlation

Example

Mixed effects model

Variance components in mixed effects model

Intraclass correlation in mixed effects model

Hypothesis

ANOVA table for mixed effects model

Estimation of fixed effects

Estimation of variance components

Estimation of intraclass correlation

Multiple comparisons in mixed effects models