

# Confidence intervals

stats6257conf 625.6 - Confidence intervals

Gunnar Stefansson

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# Interval Estimation

Recall from previous chapters: If  $X_1, \dots, X_n \sim n(\mu, \sigma^2)$  are i.i.d. random variables then

$$\bar{X} \sim n(\mu, \sigma^2/n) \quad \text{and} \quad \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim n(0, 1)$$

A method for obtaining a level  $\alpha$  confidence interval is by the so called method of inversion:

$$P\left[-z_{1-\alpha/2} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < z_{1-\alpha/2}\right] = P\left[\bar{X} - z_{1-\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}\right] = 1 - \alpha$$

So the random set

$$C(\mathbf{X}) = \left[\bar{X} - z_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}\right]$$

is a  $100(1 - \alpha)\%$  confidence interval for  $\mu$ .

stuff

## Seeking shorter confidence intervals

sometimes want to optimise the length of the CI  
(add text...)

We now want to evaluate

$$(*) \int_a^b f_Y(t) dt = 1 - \alpha$$

and find conditions which give a short confidence interval.

(\*)B

$$\int_a^b \frac{t^{r-1} e^{-nt}}{\Gamma(r)(1/n)^r} dt = 1 - \alpha$$

Could choose cutoffs  $\alpha/2$ , i.e.

$$\int_0^{\alpha/2} \frac{t^{r-1} e^{-nt}}{\Gamma(r)(1/n)^r} dt = \frac{\alpha}{2}$$

more stuff

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