# Confidence intervals

## stats6257conf 625.6 - Confidence intervals

Gunnar Stefansson

November 23, 2019

# Interval Estimation

Recall from previous chapters: If $X_1, ..., X_n \sim n(\mu, \sigma^2)$ are i.i.d. random variables then

$$\bar{X} \sim n(\mu, \sigma^2/n) \quad \text{and} \quad \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim n(0,1)$$

A method for obtaining a level $\alpha$ confidence interval is by the so called method of inversion:

$$P\left[-z_{1-\alpha/2} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < z_{1-\alpha/2}\right] = P\left[\bar{X} - z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}\right] = 1-\alpha$$

So the random set

$$C(\mathbf{X}) = \left[\bar{X} - z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{X} + z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

is a $100(1-\alpha)\%$ confidence interval for $\mu$.

# stuff

# Seeking shorter confidence intervals

sometimes want to optimise the length of the CI
(add text...)
We now want to evaluate

$$(*) \int_a^b f_Y(t)dt = 1 - \alpha$$

and find conditions which give a short confidence interval.

$$(*)B$$

$$\int_a^b \frac{t^{r-1}e^{-nt}}{\Gamma r(1/n)^r} dt = 1 - \alpha$$

Could choose cutoffs $\alpha/2$, i.e.

$$\int_0^{\alpha/2} \frac{t^{r-1}e^{-nt}}{\Gamma(r)(1/n)^r} dt = \frac{\alpha}{2}$$

# more stuff